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THE SPIN OF A QUANTUM-ENTITY AND PROBLEMS OF NON-LOCALITY.

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1 . ABSTRACT

We introduce a possible definition for the concept of non-locality in the quantum world, which seems to us a minimal operational definition, taking into account the results of actually performed experiments and reasonings about possible 'gedanken' experiments. The definition is the following: An entity is "non local" if it is possible to prepare it in a state such that it can be influenced from macroscopically separated regions of space by (macroscopically) local apparatus acting only in one (or several) of these separated regions at one time. We discuss two examples of spin superposition experiments which clearly show that quantum entities are non-local. In particular, we show that the familiar Stern-Gerlach experiment allows a nice illustration of this non-locality.

2 . INTRODUCTION.

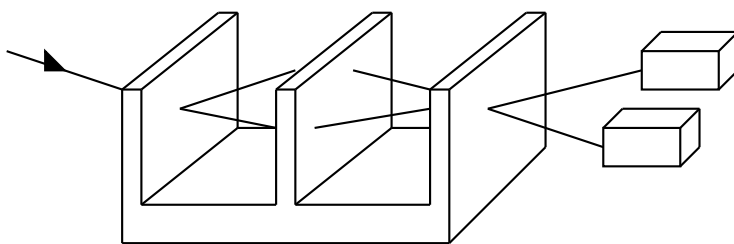
From the very beginning of the quantum theory it was clear that the so-called "quantum world" is very different from our familiar or "classical world". For a long time however the study of the quantum properties remained limited to the domain of the stationary states of microscopical physics and this has certainly greatly contributed to a comfortable attitude of "sleeping on a gentle pillow" which is still so general among the physicists ¹⁾. Recent experiments on spin correlation measurements in EPR situations have shown that the paradoxal aspect of the quantum formalism put forward in the original EPR paper can be realized (although not easily) in the laboratory. Many different analysis have meanwhile been made of these situations, and the idea of "non-locality" has been put forward for the two quantum particles in EPR situation. However even in the case of one single particle, one meets experimental situations which, when expressed in classical language, are to be called non-local. These non-local effects are usually considered as side-effects of the wave aspect of the particle. Against this rather "soft" position, we would like to suggest that non-local effects should be investigated apart from the other quantum peculiarities. In order to be definite, we shall use a minimal operational definition for the concept of non-locality,

avoiding it to be connected in a too specific way to EPR or interference situations: "*An entity is 'non local' if it is possible to prepare it in a state such that it can be influenced from macroscopically separated regions of space by (macroscopically) local apparatus acting only in one (or several) of these separated regions at one time.*"

We shall illustrate this definition by means of concrete experimental situations. In section 3 we recall one of the many neutron interferometric experiments performed by the group of Rauch. In section 4 we analyze the Stern-Gerlach experiment and notice that non-locality is already present in this situation. This point is made more explicit by a "gedanken" experiment presented in section 5. In section 6 we give some embryonic thoughts about the possible changes in the classical world image that could be envisaged to attempt to understand somewhat more about this non-locality of the quantum world.

3. THE EXPERIMENT OF RAUCH'S GROUP.

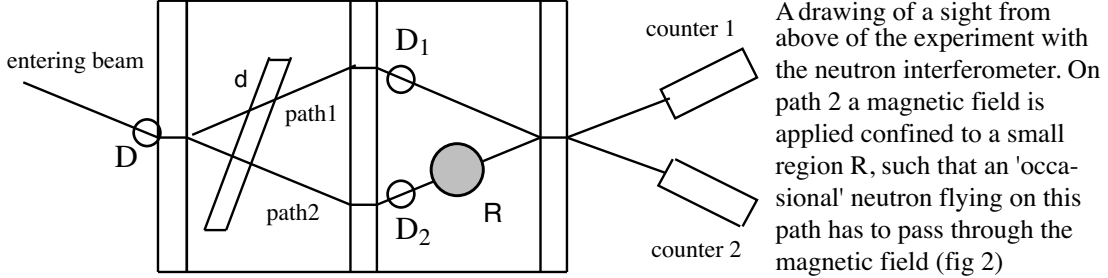
Beautiful interference experiments with thermal or ultra-cold neutrons have been carried out at the Laue-Langevin Institute in Grenoble by the group of H. Rauch²⁾. All these neutron interferometric experiments pertain to the case of self-interference, where during a certain time interval, only one neutron is inside the interferometer, if at all. We are interested in the experiment about the so called 4π -symmetry of spinors, where it is shown that if one introduces a local constant magnetic field on one of the two paths inside the interferometer with the aim of causing a Larmor precession of the spin of a neutron that "occasionally" follows this path, one has to create a 4π rotation in order to recover the original interference pattern. Let us briefly describe this experiment in order to make our point³⁾: A perfect crystal of silicon has been cut to give an interferometer of the Laue type



A perfect silicon crystal is cut in such a way to form an interferometer of the Laue type. The beam of neutrons comes in at the left, and counters measure the intensities of the outgoing beams (fig 1).

The thermal neutrons are dynamically Bragg-diffracted at the first crystal plate, then reflected at the middle plate, and finally superposed at the third plate (fig.1). Introducing a crystal plate of variable orientation (d on fig.2) one can change the relative phase of the two beams, and create an interference pattern in the detecting counters as a function of the orientation of the plate, as predicted by wave mechanics. One can modify this interference pattern by acting locally with a constant magnetic field \mathbf{B} confined to a small region R on path 2, such that an "occasional" neutron flying on this path would pass for a short period of time Δt through \mathbf{B} , and have its spin turned over an angle $\beta = \gamma \cdot \mathbf{B} \cdot \Delta t$ around the direction of this field by the Larmor precession, γ being the gyromagnetic factor of the neutron. If \mathbf{B} is taken to define the x_3 -direction, a spin in direction $\mathbf{n}(\theta, \phi) = (\sin\theta \cdot \cos\phi, \sin\theta \cdot \sin\phi, \cos\theta)$

will be transformed into a spin in direction $\mathbf{n}(\theta, \phi + \beta)$. The complex vector (spinor) $S(\theta, \phi) = (\exp(-i\phi/2) \cdot \cos\theta/2, \exp(i\phi/2) \cdot \sin\theta/2)$ of C^2 representing the spin in the quantum formalism will be changed into $S(\theta, \phi + \beta)$. We consider the case $\beta = 2\pi$; then $\mathbf{n}(\theta, \phi + 2\pi) = \mathbf{n}(\theta, \phi)$; a Larmor precession of 2π does not change the spin direction, and this is logical since if we



turn any space direction over an angle of 2π , we get the same space direction. However, since the vectors representing the spin states are vectors of C^2 , and the rotation group of C^2 is $SU(2)$, the action of such a 2π rotation is different for these vectors, in the sense that $S(\theta, \phi + 2\pi) = -S(\theta, \phi)$. There is nothing mysterious in this fact, because in quantum mechanics the state is represented by the normalized vector, or by the ray of the Hilbert space. Therefore $S(\theta, \phi)$ and $-S(\theta, \phi)$ represent the same spin state. This point is often misunderstood although it is mathematically very simple and deeply rooted in the quantum formalism, and due to the fact that $SO(3)$, the rotation group in three dimensional Euclidean space, is represented by $SU(2)$ in the group of symmetries of C^2 . This is so because one has to consider 'projective representations' of $SO(3)$ ⁴⁾, i.e. to take into account explicitly that the state of the quantum entity is represented by the ray of the Hilbert space, and not by the vector. Rauch et al. found that if $\beta = 2\pi$, the interference pattern is changed completely. Regions where before a lot of neutrons were detected, become almost empty regions, and vice versa, regions that were almost empty, now show an abundance of detected neutrons. The original interference pattern is only recovered if $\beta = 4\pi$ (or a multiple of 4π), and this is the reason why they speak of the 4π - symmetry of spinors. From our point of view, this very fascinating result shows that the picture of a "localized" neutron following a definite path is wrong. Such a "localized" neutron would only explore a narrow neighbourhood of one of the two paths, with the following alternatives: either it travels on path 1 and is not affected by the magnetic field, or, it travels on path 2 and its spin is turned over an angle 2π . Hence, in both cases, nothing happens. This of course illustrates the necessity of a wave aspect of the quantum entity. But let us now consider a quantum mechanical description of this situation, using "localized" wave-packets to represent the individual neutron, travelling alone in the apparatus. Let us firstly define what we mean by an approximately localized particle state: *a normalized wave-packet $\Psi(x_1, x_2, x_3)$ is ε -localized in the space region D if the corresponding probability integral calculated on D is larger than $1 - \varepsilon$,*

$$\int_D dx_1 dx_2 dx_3 |\Psi(x_1, x_2, x_3)|^2 > 1 - \varepsilon \quad (1)$$

which physically means that the probability of detecting the quantum entity in region D is larger than $1 - \epsilon$. Such a state will be represented by the symbol Ψ_D or alternatively by $\Psi_{D,t}$ if we like to emphasize that this happens at time t. When it enters the interferometer, at time $t=0$, the neutron is ϵ -localized in a small region D just in front of the interferometer (fig2), with a definite spin in direction $\mathbf{n}(\theta, \phi)$:

$$\Psi_{D,0} = \Psi_{D,0}(x_1, x_2, x_3) \otimes S(\theta, \phi) \quad (2)$$

The experimental situation is a bit complicated to allow an exact treatment of the Schrödinger equation, but this is not really necessary in order to understand what is going on. The initial wave packet evolves into two separated wave packets the centres of which follow approximately the paths previously described. The state of the neutron at a certain time t, when it is inside the apparatus, is described by a wave function

$$\Psi_t = \Psi_{D_1,t} + \Psi_{D_2,t} \quad (3)$$

$$\text{where } \Psi_{D_1,t} = \Psi_{D_1}(x_1, x_2, x_3) \otimes S^1(\theta, \phi) \quad (4)$$

$$\text{and } \Psi_{D_2,t} = \Psi_{D_2}(x_1, x_2, x_3) \otimes S^2(\theta, \phi)$$

$\Psi_{D_1,t}$ is a product state, the space part of which is represented by the wave packet Ψ_{D_1} , ϵ -localized in a region D_1 on path 1 (analogous $\Psi_{D_2,t}$). The precise form of the wave packets is rather irrelevant for our purpose, and therefore we forget about the deformation of the wave-packets ϵ -localized in regions D_1 and D_2 , and consider instead these regions as evolving along the paths 1 and 2, respectively, when the time elapses (fig2). Rauch applies a magnetic field in a region R which is crossed by the region D_2 during some time interval ($t_0, t_0 + \Delta t$) causing a Larmor precession of the spinor S^2 , so that only one of the two parts of the wave function of the neutron is affected by this magnetic field. If the experimental arrangement is such that $\beta = 2\pi$, then the spinor S^2 changes its sign, which means that, comparing two experimental situations, the first one without the magnetic field (or B inactive) and the second one with the magnetic field as here above described (B active), one gets after the crossing of the region R:

$$\{\Psi_{D_2}\}_{B \text{ active}} = - \{\Psi_{D_2}\}_{B \text{ inactive}} \quad (5)$$

But then the 'true' wave function Ψ_t representing the state of the neutron undergoes the following changes: Before the time t_0 where the movable region D_2 reaches the region R, the wave function remains the same in both cases (i.e. B active or inactive). After the time $t_0 + \Delta t$ where the movable region D_2 has fully crossed the region R, the wave function becomes when B is active :

$$\begin{aligned} \{\Psi\}_{B \text{ active}} &= \{\Psi_{D_1} + \Psi_{D_2}\}_{B \text{ active}} \\ &= \{\Psi_{D_1} - \Psi_{D_2}\}_{B \text{ inactive}} \end{aligned} \quad (6)$$

which is of course deeply different from $\{\Psi\}_{B \text{ inactive}}$ at the same time. The change in the interference pattern that is observed by the experimenters is nothing else than the statistical realization of this elementary change, and it is indeed explained in that way in their publications. We only want to insist, on purpose, on the local character of the action which changes the state of one single neutron (even if the experimental observation of this fact is statistical).

It is clear that one can make the same reasoning about an experiment placing the region R on the path of the wave-packet number 1.

This means that *the single neutron can be influenced from macroscopically separated regions of space by (macroscopically) local apparatus acting only in one of these separated regions at one time*. This is exactly our criterion of non-locality.

To emphasize the amazing character of the effect of 'de-localization' of one neutron realized in the experiment, we would like to make a rough comparison based on a scaling which respects the relative orders of magnitude. The longitudinal coherence length of these neutrons, experimentally shown to be of the order of 20 Å, is a good measure of the size of a wave packet and hence in a certain sense an 'upper bound' for the 'size' of the neutron. Since the regions D, D₁, and D₂ are represented on the figures by means of spots of more or less 0.5 cm diameter, there is a scaling factor of 0.25·10⁷. On the other hand, the incoming neutrons follow each other at an average distance of some 300 m, the real distance between the two paths is of the order of 5 cm, and the region R is a square with sides equal to 0.15 cm. Hence, in an "on scale" drawing, neutrons like little balls of 0.5 cm diameter would be shot having an average distance of 750.000 km between them (i.e. nearly 2 times the distance moon earth), the two paths would be separated by 125 km and the region R would be 4 km squared. At this scale, the experiment literally means that we can de-localize one single object for which we normally would imagine it to be enclosed in a sphere of 0.5 cm, over a distance of 125 km, in the sense that this 'one' object can be influenced from two space regions that are like small villages of a few km² some 125 km apart. It is clear that this 'non local Rauch effect', is not some kind of 'peculiar' property of 'neutrons' or 'magnetic fields' and the conclusion about the non-locality should be accepted also for other quantum-entities, and also for other force fields. Notice however that interference patterns are always the result of a statistical sampling of individual cases. Hence, to be able to draw more direct conclusions about single entities, it is probably better to discuss experiments where the effect of a local perturbation can be seen on each individual entity or equivalently, experiments where the statistical prediction turns out to be a certainty. We want to consider now such an experiment, the principle of which is quite analogue to the one of Rauch, in which exactly the same effect of de-localization of one quantum entity happens and can in principle be observed on each single entity.

4. THE STERN-GERLACH EXPERIMENT.

We now want to discuss the old Stern-Gerlach experiment, which in fact can be considered as the true discovery of the electron spin ⁵⁾. Stern and Gerlach prepared a beam of silver atoms travelling with a rather well defined velocity, in a certain space direction **b** (beam). The beam enters a strong 'inhomogeneous' magnetic field, with a strong constant part along a space direction **a**, orthogonal to **b**, and also having a strong 'gradient'. When the atoms of the beam leave the magnet, they are detected on a screen, placed orthogonal to **b**. Stern and Gerlach found that the individual atoms are detected in two spots on the screen:

one spot, 'upwards', in the \mathbf{a} direction, and another spot 'downwards', in the $-\mathbf{a}$ direction. In a time when modern quantum mechanics did not yet exist, but the idea of quantization was already very common, they interpreted their result as an indication that the silver atoms have a 'quantized magnetic moment'. They had in mind the following 'classical picture' of the mechanism taking place during the experiment : The silver atom is imagined to be a classical particle of mass M with a magnetic moment μ due to its orbiting valence electron. When it enters the region of the magnetic field, it interacts with this magnetic field in essentially a classical way: the magnetic moment vector $\boldsymbol{\mu}$ precesses rapidly around the \mathbf{a} direction and the whole atom undergoes a force due to the coupling of the magnetic moment to the gradient of the field. The classical calculations show that the field gradient causes a deflection of the impact of the beam on the screen which extends on a full interval whose extremities correspond to the spots detected in the experiment. This 'quantized' orientation of the angular momentum remains up to our days the basic concept that physicists use when they try 'to represent', 'to picture', or 'to talk about', the spin of a quantum entity. Let us now consider the modern quantum mechanical description of this situation. Unlike the case of the Rauch experiment, one can calculate in detail the quantum mechanical evolution of one single atom in the Stern-Gerlach experiment, by solving the time dependent Schrödinger-Pauli equation with an interaction potential energy equal to $-\boldsymbol{\mu} \cdot (\boldsymbol{\sigma} \cdot \mathbf{B})$, where μ is the magnetic moment of the atom, $\boldsymbol{\sigma} = (\sigma_1, \sigma_2, \sigma_3)$ are the Pauli matrices, and \mathbf{B} is the magnetic field (limited in practice to a small region along the magnet), with an appropriate initial condition.

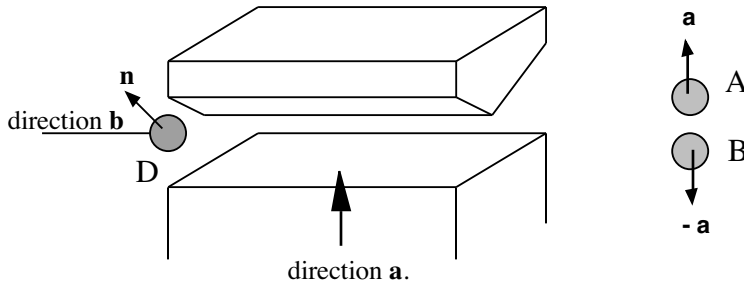
$$\begin{aligned} i\hbar \frac{\partial}{\partial t} \Psi(t) &= -\frac{\hbar^2}{2M} \Delta \Psi(t) - \boldsymbol{\mu} \cdot \boldsymbol{\sigma} \cdot \mathbf{B} \cdot \Psi(t) \\ \Psi(0) &= \Psi_0 \end{aligned} \quad (7)$$

We consider the situation where an atom with a well defined spin direction enters the Stern Gerlach magnet, at time $t = 0$. Its state is then represented by a normalized vector Ψ_0 , of the form $\Psi_D(\mathbf{x}) \otimes (\alpha, \beta)$, where $\Psi_D(\mathbf{x})$ is a wave-packet ε -localized in region D , localized at the entrance of the Stern-Gerlach apparatus (fig3), and $(\alpha, \beta) = S(\theta, \phi) = (\cos(\theta/2) \exp(-i\phi/2), \sin(\theta/2) \exp(i\phi/2))$ representing the spin direction $\mathbf{n}(\theta, \phi)$ defined with respect to the directions $\mathbf{a} = (0,0,1)$ and $\mathbf{b} = (1,0,0)$. It is remarkable, taking into account the importance of the Stern-Gerlach experiment and the apparent simplicity of the problem, that nowhere in the literature a sound quantum mechanical treatment exists. Most treatments consider incorrect magnetic fields (not satisfying the Maxwell equations), or they do not give any satisfactory explanation for the approximations used to construct a 'solution' of the equation. Therefore we have investigated again the quantum mechanical treatment of the Stern-Gerlach experiment, trying to avoid the weakness of the earlier attempts ⁶). We consider the case of an incoming atom represented by a small three dimensional gaussian wave packet with a mean velocity v_0 in the $\mathbf{b} = (1, 0, 0)$ direction. We follow the evolution of the wave packet, when it travels through the magnet and analyze in which way we can construct a solution, making the appropriate physically significant approximations. This allows us to compute the wave

function at a time τ when the atom leaves the magnet and we find that it is then clearly separated into two wave packets. We then follow the further evolution of these free wave packets until they reach the screen. The result is rather complex to write down explicitly but for the present discussion a detailed expression is not necessary. Enough is to say that the wave function that leaves the magnetic field is of the form

$$\Psi_{\text{out}} = \alpha \cdot \Psi_A \otimes (1,0) + \beta \cdot \Psi_B \otimes (0,1) \quad (8)$$

where A and B are two separated regions in space (fig3), and where we keep the same notations as previously to indicate the ε -localization of a wave packet in a region of space.



The region D is the space region where the incoming atom is ε -localized, and the vector \mathbf{n} is the space direction of the spin of the incoming atom. The regions A and B are the space-regions where the component wave-functions of the superposition wave function representing the state of the outgoing atom, are ε -localized.

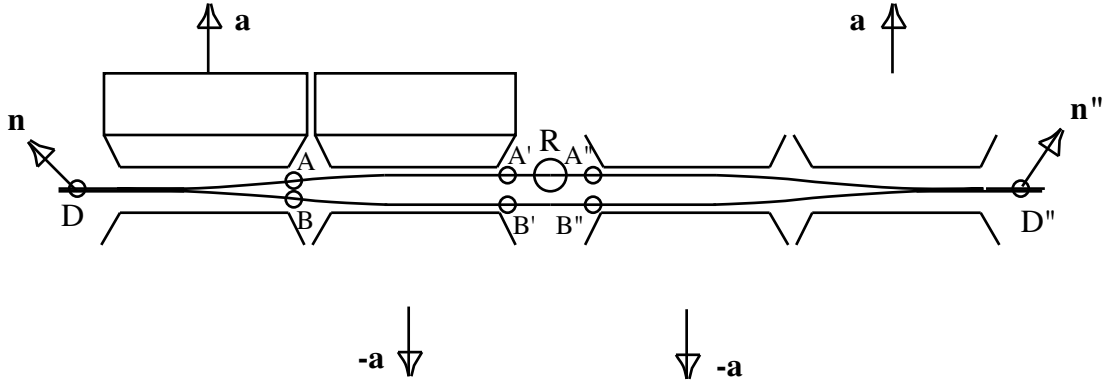
These component wave functions appear in the superposition with spin-vectors corresponding to respectively space direction \mathbf{a} and $-\mathbf{a}$. The complete wave function representing the state of the outgoing atom does not have a spin corresponding to a fixed space direction (fig 3).

Remember that in all the fig. of this paper, we do not make a drawing of the quantum entities but only of the regions where they are ε -localized ; the arrows attached to these regions are mathematical directions which only occasionally indicate the spin direction of the atom (this is the case for region D, this is not the case for regions A and B). We want to analyse now the physical meaning of the state represented by this vector Ψ_{out} , the state of the single atom when it leaves the magnet before it is detected. Most physicists agree on the fact that the purely classical model of a particle having a 'quantized' rapidly precessing magnetic moment, is not a correct model for the 'spin' of a quantum-entity. Through the years some kind of 'semi-classical' picture has developed. The final state vector Ψ_{out} is often implicitly interpreted in the following way : When an atom enters the Stern-Gerlach apparatus, it either really 'flips up', and flies on the 'upper' beam, and it is then *de facto* in a state represented by the vector $\Psi_A \otimes (1, 0)$, or it 'flips down' and flies on the 'lower' beam, and is then *de facto* in a state represented by the vector $\Psi_B \otimes (0,1)$. The very fact that, according to the calculations of quantum mechanics, the final state is represented by the vector Ψ_{out} is interpreted as reflecting our lack of knowledge about which of the two possibilities the atom chooses. If this 'semi-classical' interpretation would be correct, then the atoms that fly through a Stern-Gerlach magnet have always a well defined spin direction, and are always ε -localized in one of the separated regions A or B. Of course, this is not true from first principles of quantum mechanics and also from the results of the Rauch experiment that one can translate *mutatis-mutandis* to the Stern-Gerlach case. But, if the 'semi-classical' interpretation is wrong, what else can we propose ? Let us try to describe a possible experiment

of the Rauch type which sustains the thesis that atoms coming out the magnet are in a non-local state.

5. THE NON-LOCALITY OF AN ATOM LEAVING THE STERN GERLACH APPARATUS.

Experiments that consist in combining different Stern-Gerlach apparatus to illustrate the amazing properties of the superposition principle have been considered in the past by many authors and are described in various text-books ⁷⁾. We consider such an experiment on purpose of showing that an atom in the state Ψ_{out} is non-local in the sense of our definition. It is true that the experiment we shall describe is probably impossible to realize now and perhaps for ever because one cannot control the inhomogeneous magnetic fields in the different magnets of the set-up with a sufficient accuracy ⁸⁾. But we are confident in the results because of the great similarity with the work of Rauch et al. with neutrons. And this 'gedanken' experiment has the advantage of working directly on one single atom or equivalently with the statistical weight of certainty. One sends a beam of atoms flying in direction $(1, 0, 0)$, with a definite spin direction along $\mathbf{n}(\theta, \phi)$ into a first Stern-Gerlach magnet and a magnetic field in direction $\mathbf{a} = (0, 0, 1)$. Then one puts right behind it a second Stern-Gerlach magnet identical to the first one except for the field direction which is opposite: $-\mathbf{a} = (0, 0, -1)$. This second magnet will converge the two beams (to describe the experiment we use the classical language) in such a way that they become parallel (fig4). Then one puts a third Stern-Gerlach magnet, identical to the second one, which will incline the beams



A representation of the experiment, combining four Stern-Gerlach magnets, such that the original beam is recombined. In region R we apply a constant magnetic field, and turn the spin over an angle of 2π . The spin of an outgoing atom has then been turned over an angle of π around the \mathbf{a} direction (fig 4).

towards each other. Finally one puts a fourth Stern-Gerlach magnet, identical to the first one, which will have the effect of reconstituting the two beams into a single one which is in principle identical to the original incoming beam. In a region R one applies a constant magnetic field, such that the spin can be turned by the Larmor precession, over different angles. One makes the experimental arrangement in such a way that the spin will turn over an angle of 2π . Notice that because such a rotation is equivalent to the identity, the axis of rotation (i.e. the direction of the magnetic field) is irrelevant for the results of the experiment. Let us

now analyze the quantum mechanical description, to see what will happen. As before, the state at time $t = 0$ is represented by a normalized vector Ψ_0 of the form $\Psi_D(x) \otimes (\alpha, \beta)$ (see section 3). Again we have to solve the Schrödinger-Pauli equation (7) for the evolution of this vector through the first Stern-Gerlach magnet, such that the state of the atom coming out of the first magnet is given by Ψ_{out} (see(8)), where A and B are separated regions (see section 3 and fig. 4). The second Stern-Gerlach magnet will not change anything to the spin components, since they are $(1, 0)$ and $(0, 1)$, but will influence the direction of flight of the two wave packets Ψ_A and Ψ_B , so that they become parallel at its end, so that the atom coming out of the second magnet will be in a state $\alpha \cdot \Psi_{A'} \otimes (1,0) + \beta \cdot \Psi_{B'} \otimes (0,1) = \Psi'_{out}$ (see fig. 4). If we now apply a constant magnetic field in the region R localized on the upper beam (fig4), then only the spin of the partial vector $\alpha \cdot \Psi_{A'} \otimes (1,0)$ will be influenced. If it is turned over an angle of 2π , then the spinor $(1, 0)$ will be changed into the spinor $(-1,0)$, no matter what the direction of the magnetic field applied in the region R can be. Of course, as we recall it in our reasonings about the Rauch experiment, these two spinors represent the same state. So by the effect of the Larmor rotation we find at the entry of the third Stern-Gerlach magnet a state of the atom represented by a vector $\alpha \cdot \Psi_{A''} \otimes (-1, 0) + \beta \cdot \Psi_{B''} \otimes (0, 1) = (-\alpha) \cdot \Psi_{A''} \otimes (1, 0) + \beta \cdot \Psi_{B''} \otimes (0, 1) = \Psi''_{out}$, where A'' and B'' are again separated regions in space (fig 4). To proceed and find the quantum mechanical effect of the two last Stern-Gerlach magnets, we notice that the Schrödinger dynamics is invariant for time reversal. So instead of trying to calculate the evolution up to the end of the fourth magnet of a state which is of the form Ψ''_{out} at the entrance of the third one, we can inverse the problem and consider the question : which state of the form $\Psi_{D''}(x) \otimes (\gamma, \delta)$ at the end of the fourth magnet will give rise to the state Ψ''_{out} at the entrance of the third magnet by a time reversed Schrödinger evolution? The answer to this question is immediately given by the first part of the calculation, and is of course $\Psi_{D''}(x) \otimes (-\alpha, \beta)$.

Hence if the spin of the incoming atom is in direction $\mathbf{n}(\theta, \phi)$, then using in the region R a constant magnetic field of appropriate magnitude but arbitrary direction, the spin state $\mathbf{n}''(\theta, \phi)$ of the outgoing atom has turned over an angle π around the direction \mathbf{a} defined by the Stern-Gerlach magnets. In the classical picture of spinning particles this magnetic field in region R would have turned the spin over 2π , which means that each atom would leave the apparatus exactly in the same state as it enters. The same result would be found in the semi-classical picture. If we complete the experiment by a Stern-Gerlach measurement of the orientation of the spin of the final atom, there is no doubt that it will fully confirm the standard quantum mechanics prediction of a rotation of π ! In this sense it is clear that the experiment shows a realization of our definition of non-locality for quantum entities.

6. CONCLUSION.

We want to ask now again what the physical meaning is of the state of a quantum-entity of the form $\alpha \cdot \Psi_A \otimes (1,0) + \beta \cdot \Psi_B \otimes (0,1)$. In our opinion the following statement can be made : The state $\alpha \cdot \Psi_A \otimes (1,0) + \beta \cdot \Psi_B \otimes (0,1)$ represents one quantum entity, neither localized in region A with spin in the direction $\mathbf{n} = \mathbf{a}$, nor localized in region B with spin in

direction $\mathbf{n} = -\mathbf{a}$. Moreover *this one quantum-entity can be 'influenced' from two separated regions A and B by local apparatus acting only in one of these regions at one time*. This does not allow to conclude that this one quantum-entity is at two places at the same time and not between. We seriously consider the possibility that our 'classical' conception of three dimensional Euclidean space and localized entities is just not convenient to picture what really happens. Probably "the human part of the construction of the reality" (see H. Poincaré, ref. 9) is to be reconsidered in order to begin to approach the quantum mechanical non-locality. In ¹⁰⁾ a similar proposal was made in relation with the problematic around the EPR situation, and on the hand of some example it was shown that it is not impossible, 'in principle', to imagine such an approach, that could lead to a human understanding of the quantum non-locality.

7. ACKNOWLEDGEMENT.

We want to thank J. Broekaert for presenting this paper at the conference.

8. REFERENCES.

1. This expression seems to have been used by Einstein in a letter to Schrödinger (See the article of Mermin, N.D., Physics Today, April, 9 (1989).
2. These experiments are described in great details in several publications of members of this group. One find a sufficient list of references in Rauch, H., '*Neutron interferometric tests of quantum mechanics*', Helv. Phys. Acta, 61, 589 (1988).
3. Rauch H., Zeilinger A., Badurek G., Wilfing A., Banspiess W., Bonse U., '*Verification of Coherent Spinor Rotation of Fermions*', Phys.Lett. 54A, 425 (1975).
4. Bargmann, V., Ann. Math., 59, 1, (1954), see also, Hammermesh, M., "*Group theory and its applications to physical problems*", Addison Wesley, Reading, Mass.,(1962), and Wigner, E.P., "*Group theory and its applications to quantum mechanics and atomic spectra*", Academic Press, New York, (1959).
5. Gerlach,W. and Stern,O., "*Der experimentelle Nachweis der Richtungsquantelung im Magnetfeld*", Zeitschrift für Physik, 9,349, (1922).
6. Aerts, D. and Reignier, J., " *A quantum mechanical treatment of the Stern Gerlach experiment* ", in preparation.
7. See for instance '*The Feynman Lectures on Physics*', Vol. III, Ch. 5,6.
8. Schwinger J., Scully M.O., Englert B.-G., '*Is spin coherence like Humpty Dumpty ?*' Zeits. f. Phys. D., 10, 135 (1988).
9. About this question of the human part of the construction of space it is still very interesting to read again the small book of H. Poincaré "*La science et l'hypothèse* "
10. Aerts D., "*An attempt to imagine parts of the reality of the micro-world*", in the proceedings of the conference '*Problems in Quantum Physics ; Gdansk '89*' to be published by World Scientific Publishing Company, Singapore.