Published as: Aerts, D., 1991, "A macroscopical classical laboratory situation with only macroscopical classical entities giving rise to a quantum mechanical probability model", in *Quantum Probability and Related Topics, volume VI*, ed. Accardi, L., World Scientific Publishing Company, Singapore, 75 - 85.

A macroscopical classical laboratory situation with only macroscopical classical entities giving rise to a quantum mechanical description.

Diederik Aerts,
TENA, Vrije Universiteit Brussel,
Pleinlaan 2, 1050 Brussels, Belgium.

Abstract : We propose a macroscopical classical physical entity, giving a detailed description of the preparation apparatuses and the preparations (states) of this entity. We consider experiments that can be performed on the entity, and give a detailed description of the measurement apparatuses, and the measurements used in these experiments. We investigate the collection of probabilities for the outcomes of the measurements the entity being prepared in a given state. Therefore we use the ordinary meaning of probability as approximate relative frequency of repeated experiments, hence experiments consisting of equivalent measurements on equivalently prepared entities. We show that the collection of probabilities that results in this way for our macroscopical entity is the same as the collection of probabilities for the outcomes of the Stern-Gerlach spin measurements on a spin 1/2 quantum entity prepared in a given spin state. By analyzing in which way this purely classical physical situation gives rise to a quantum probability model, we propose an explanation for the non classical probability structure of the quantum probability model. We conclude by showing that this explanation is plausible from a physical point of view, and if accepted makes disappear a lot of the paradoxical nature of the quantum formalism, in the sense that the quantum probabilities do not have to be interpreted any more as 'ontological' or 'intrinsically' present in nature itself.

1. Introduction.

The aim of this paper is not to take part in the discussion of whether a hidden variable theory can replace quantum mechanics. We know that already earlier, hidden variable models for the spin have been constructed, on a purely mathematical basis, without giving any meaning to the hidden variables. This is not what we will do. We will describe a physical 'laboratory' situation, using a macroscopical classical entity, giving a description of the states in which this entity can be prepared, and of the experiments on this entity, such that the approximate relative frequencies of such repeated experiments give rise to a collection of probabilities equivalent to the collection of probabilities that is found for the outcomes of the Stern-Gerlach spin measurements on a spin 1/2 quantum entity prepared in a given spin state. So we do not want to 'imitate' this quantum entity by introducing a mathematical model, but by giving a real physical situation. That is the reason why we have to introduce very explicitly the concepts 'experiment' 'preparation' and 'measurement' and give a detailed description of the preparation apparatuses and the measuring apparatuses. In section 2 we will introduce carefully these concepts for the spin of a spin 1/2 quantum entity. In section 3 we will do this for the macroscopical classical entity. In section 4 we will try to analyze the meaning of the possibility of realizing such macroscopical situations for the interpretation of quantum mechanics.

2. The spin of a spin 1/2 particle.

Theories are connected with reality by means of a procedure that has been called an experiment. Instead of trying to stipulate in general all the steps of this procedure, we will introduce the necessary aspects by means of the example of the spin measurements of the spin of a spin 1/2 entity, and then in next section by means of the macroscopical classical example. A general exposition of this procedure can be found in [1]. Let us now introduce the different primitive elements for an adequate description of experiments on the spin of a spin 1/2 quantum entity.

Q1) The measurement apparatus MA(Q, a) and the measurements m(Q, a) : The measurement apparatus MA(Q, a) consists of a Stern-Gerlach magnet [2], which is a strong 'inhomogeneous' magnetic field, that has a strong constant part along the space direction **a**, and also a strong 'gradient', and a detection screen placed behind the magnet. The construction of this apparatus is very complicated, involving a lot of physical manipulations that are not taken into account, and not described. The 'only part' of its construction that is finally retained in the theoretical representation of the measurement is the space direction **a** of the constant part of its magnetic field. The measurement m(Q, **a**) consists of shooting a beam of quantum entities (with spin 1/2) into the magnetic field along a space direction **x** orthogonal to **a**. Two spots are detected on the screen, one spot upwards and one spot downwards of where the **x** direction of flight intersects the screen. If a quantum entity of the beam is detected in the upper spot of the screen, we call this outcome "**a**-up" and if it is detected in the lower spot of the screen,we call this outcome "**a**-down" of the measurement m(Q, **a**).

Q2) The preparation apparatus PA(Q, b) and the preparations p(Q, b) : The preparation apparatus PA(Q, b) again consists of a Stern-Gerlach magnet, along the space direction **b**. But now we place a half screen behind the magnet, that blocks of the lower region where normally the 'down' spot would appear on the total screen, and leave open the upper region. The preparation p(Q, **b**) consists of constructing a narrow beam of quantum entities travelling with a rather well defined velocity in the **x** direction of space. For example in the original Stern-Gerlach experiment [2] silver atoms are used produced by an oven at 500^{o}K, and the beam can be defined by two successive diaphragms of 0.1 mm separated by 10 cm and carefully set along the **x** direction.This results in an average classical velocity v_b of more or less 400 m/sec, while the other components of the velocity are very small of the order of 40 cm/sec. Hence the quantum entities are well 'separated' and during a passage through one of the apparatuses only one quantum entity at once will be inside, such that is acted on individual quantum entities, one after the other. Then the beam enters the preparation apparatus PA(Q, **b**). Some of the quantum entities will be absorbed by the half screen that blocks of the lower region, and some of the quantum entities will fly on. This collection of actions we will call preparation p(Q, **b**). Suppose now that after the preparation p(Q, **b**) we perform a measurement m(Q, **b**). Then not two spots, but only one spot, a little bit more upwards is detected on the screen of the measurement apparatus MA(Q, **b**). This shows that preparation p(Q, **b**) has produced a kind of 'purified' beam, consisting of quantum entities that 'have' the 'property' of 'being detected in the upper spot of the screen' when a measurement m(Q, **b**) would be performed. We will give a 'name' to this property, and say that 'the quantum entities have their spin 'up' in the space direction **b**'. So by means of preparation p(Q, **b**) a quantum entity can be **prepared** with its spin in space direction **b**. We want to remark that the preparation p(Q, **b**) does not mean neither more nor less than what we come to explain.

Q3) The experiments e(Q, a, b) : The experiment e(Q, **a, b**) consists of preparation p(Q, **b**) followed by measurement m(Q, **a**).

Q4) The probabilities P(Q, a, b) and P(Q, -a, b) : To come to the probabilities we must make repeated experiments e(Q, **a**, **b**) consisting of equivalent preparations p(Q, **b**) and equivalent measurements m(Q, **a**). We can count the number N(**a**-up) of outcomes "**a**-up" or the number N(**a**-down) of outcomes "**a**-down" of the measurements, and divide by the total number N of quantum entities that have participated in the repeated experiments. The numbers ν(**a**-up) and ν(**a**-down) between 0 and 1 that we get in this way are called the relative frequencies of the corresponding outcomes. If these relative frequencies approximate real numbers between 0 and 1 if N goes to infinity (which means if we keep repeating the same experiment) , then we call these real numbers the probabilities P(**a**-up) and P(**a**-down), and we interpret them as the probabilities for the corresponding outcomes to occur if the corresponding preparation is made. In the concrete situation of the preparations and measurements in relation with the quantum system of spin 1/2, we can introduce the following probabilities :

P(Q, **a, b**) = the probability that if a quantum entity is prepared with the preparation p(Q**, b**), and the measurement m(Q**, a**) is performed, the outcome "**a**-up" will occur or hence the probability that an experiment e(Q, **a, b**) will make occur an outcome "**a**-up".

P(Q, -**a, b**) = the probability that if a quantum entity is prepared with the preparation p(Q**, b**), and the measurement m(Q**, a**) is performed, the outcome "**a**-down" will occur or hence the probability that an experiment e(Q, **a, b**) will make occur an outcome "**a**-down".

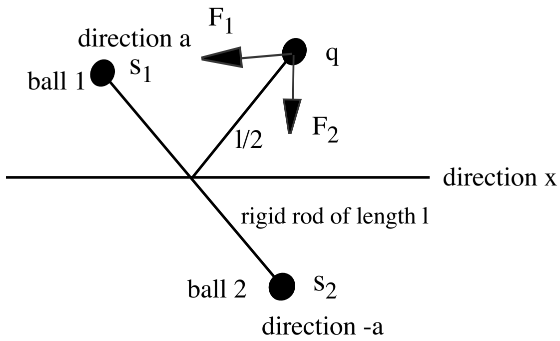
To find the probabilities we must go to the laboratory and perform the Stern-Gerlach experiment (that is the rule of the game). This is exactly what Stern and Gerlach have done, and they proved in this way the quantization of the magnetic moment of an atom. At that time it was experimentally impossible to find the probabilities (we remark that at the time of the experiment the mathematical formalism of quantum theory did not yet exist). The Stern-Gerlach experiment remains a very difficult experiment, and even actually can only be performed with a great deal of inaccuracy as to the values of the probabilities that we have considered. A detailed description of the difficulties related with its experimental execution can be found in [3]. By turning the Stern-Gerlach apparatus oriented along **a**, we can see that the intensity measured in the spots is changed corresponding to a law related only to the angle between **a** and **b**, and more concretely that if we take P(Q, **a, b**) = $\cos^2(\gamma/2)$ where γ is the angle between the space directions **a** and **b**, we are in relative good agreement with the experimental findings.

Q5) The predictions of the quantum mechanical model of the situation : Quantum mechanics gives us a nice mathematical representation of the preparations (states) and measurements that we have come to describe in relation with the spin of a spin 1/2 entity, and also of the collection of the probabilities that we have introduced, which is the well known quantum description of the property spin, in the two dimensional complex Hilbert space. This model predicts indeed P(Q, **a, b**) = $\cos^2(\gamma/2)$ where γ is the angle between the space directions **a** and **b**.

3. The macroscopical situation with a spin 1/2 probability model.

The classical macroscopical spin model that we will present in this section has been presented in [4] with the aim of giving a possible explanation for the the non-classical character of the quantum probabilities. It is shown in [4] that a lack of knowledge about the measurements on a physical entity gives rise to a collection of probabilities connected to this entity, that is non-classical. It is also shown that the non classical probability calculus of quantum mechanics can be interpreted as being the result of a lack of knowledge about the measurements. In this paper we want to introduce essentially the same example, but now we describe in much more detail the measurement apparatus, the preparation apparatus, the measurements, and the preparations, such that it is shown that the relative frequencies of repeated experiments of this classical situation indeed leads to the same collection of probabilities as the one of the spin of a spin 1/2 quantum entity. Let us introduce the macroscopical classical model.

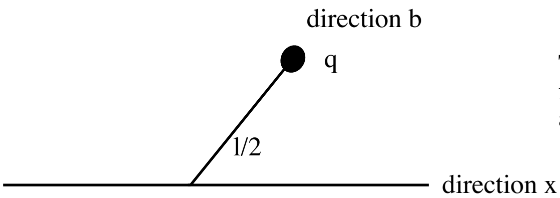
C1) The measurement apparatus MA(C, a) and the measurements m(C, a) : We give a detailed description of the construction of the measurement apparatus MA(C, **a**). We have a rigid rod constituted of a non-conducting material (for example some plastic) of a certain length l (see fig 1). At the end-points of the rod are two little iron balls, ball 1 and ball 2. In the laboratory where our experiment will be performed we have a battery at our disposal, that contains a certain fixed amount of negative charge s. There is a mechanism that brings both of the iron balls during a certain time in contact with the battery, such they get charged. As a result ball 1 shall be charged with a certain charge s_1 and ball 2 with a charge $s - s_1 = s_2$. Then the balls get uncharged again. And immediately after they get charged again, and then uncharged again. And so on. The rigid rod is placed fixed in the laboratory such that ball 1 is in space direction **a**, and ball 2 in space direction **-a** in a plane orthogonal to some fixed direction **x**. The physical entities that we consider are little iron balls, positively charged with a fixed charge q. They can be put in the neighbourhood of the measurement apparatus, and connected to it by a non conducting rigid rod of length l/2. The measurement m(C, **a**) consists of letting go the positive charge q. It will be attracted by the



The measuring apparatus MA(C, a) consists of a rigid rod of lenght l with at the endpoints two iron balls, ball 1 and ball 2, alternatively charged and decharged by an amount of negative charge s, such that ball 1 gets one part s_1 of the charge and ball 2 the other part $s_2 = s - s_1$. The measurement m(C, a) consists of connecting a positively charged ball q at the centre of the rigid rod of the measuring apparatus by means of a rigid rod of lenght l/2. The charge q will be captured by one of the two negative charges. If it is captured by s_1 then we give outcome "a-up" to measurement m(C, a), if it is captured by s_2 then we give outcome "a-down" to the measurement m(C, a) (fig 1).

two negative charges of the measuring apparatus by Coulomb forces F_1 and F_2. We suppose that this happens in a viscous medium, such that under the influence of friction, finally the positively charged ball q will end up at one of the balls of the measuring apparatus. If it ends up at ball 1 we give the outcome "**a**-up", and if it ends up at ball 2 we give the outcome "**a**-down" for the measurement m(C, **a**).

C2) The preparation apparatus PA(C, b) and the preparations p(C, b) : To make the analogy with the spin complete, we suppose that we cannot 'directly' manipulate the charge q (as we cannot directly manipulate a quantum entities spin). Hence the preparation of the state of q has to be done by means of a preparation apparatus PA(C, **b**). Exactly as for the spin case, the preparation apparatus is slightly different from the measurement apparatus. We set at work the part of the measuring apparatus MA(C, **b**) which is the rigid rod with its two charged balls 1 and 2 on the ball q, but only conserve these balls q that have been captured by ball 1 of the measuring apparatus (the balls q that are captured by ball 2 are considered to be annihilated). Hence after the preparation, with apparatus PA(C, **b**), the charge q is at a location in direction b, and length l/2 from the centre of the measurement apparatus rod.We call this preparation p(C, **b**) (see fig 2).



The preparation p(C, b) by means of the preparation apparatus PA(C, b) (fig 2).

C3) The experiments e(C, a, b) : The experiment e(C, **a, b**) consists of a preparation p(C, **b**) followed by a measurement m(C, **a**).

C4) The probabilities P(C, a, b) and P(C, -a, b) : We can now start making the repeated experiments. In relation with the preparation p(C, **b**) and afterwards the measurements m(C, **a**), we can count the number N(**a**-up) of outcomes "**a**-up" or the number N(**a**-down) of outcomes "**a**-down" and divide by the total number N of classical entities that have participated in the repeated experiments. If the relative frequencies ν(**a**-up) and ν(**a**-down) approximate real numbers between 0 and 1 if N goes to infinity, then we call these real numbers the probabilities P(**a**-up) and P(**a**-down). We can introduce the following probabilities :

P(C, **a, b**) = the probability that if the classical entity C is prepared with the preparation p(C**, b**), and the measurement m(C**, a**) is performed, the outcome "**a**-up" will occur and hence the probability that the experiment e(C, **a, b**) makes occur outcome "**a**-up".
P(C, -**a, b**) = the probability that if the classical entity C is prepared with the preparation p(C**, b**), and the measurement m(C**, a**) is performed, the outcome "**a**-down" will occur and hence the probability that the experiment e(C, **a, b**) makes occur outcome "**a**-down".
To determine these probabilities, we must go to a laboratory and perform the repeated experiments, and then see what we find for the relative frequencies. If we do this we find that they will depend on the angle between the two space directions **a** and **b**, and if we take P(C, **a, b**) = $\cos^2(\gamma/2)$ where γ is the angle between the space directions **a** and **b**, we are in good agreement with the experimental findings.

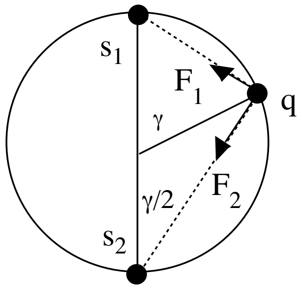
C5) The classical mechanics predictions for the probabilities.

We have created two experimental situations, one using quantum microscopic entities, and one using classical macroscopical entities, that lead experimentally to an equivalent collection of probabilities. The probabilities of the quantum example can be derived also theoretically, using the quantum mechanical description of the Stern-Gerlach measurement situation. But it must be pointed out again that this quantum calculation does not give any specification on the physical mechanism by which the quantum measuring apparatus MA(Q, **a**) affects the quantum entity in preparation p(Q, **b**) to lead to one of the possible outcomes. This mechanism, taking into account the enormous complexity of the measuring apparatus MA(Q, **a**), is probably very complicated, and full of hidden randomness. For our classical example we know the mechanism, and hence can propose a classical mechanics model for it. Of course, like every theoretical model (also the quantum model), this model is an idealization of the reality happening in the laboratory. It can be interpreted as a kind of simple model for the working of the quantum apparatus MA(Q, **a**). Let us regard the measurement situation of our classical macroscopical example a little bit closer, and see which model we can propose. The three charges are located in a plane, the positive charge q, prepared by p(C, **b**), in a point indicated by the direction **b**, and the two negative charges of M(C, **a**) in diametrically opposed points indicated by the directions **a** and **-a** (see fig 3). Let us call γ the angle between the two space directions **a** and **b**. The forces F_1 and F_2 are the Coulomb forces, hence

$$|F_1| = \frac{s_1 \cdot q}{\pi\varepsilon_0 l^2 \sin^2(\gamma/2)} \quad |F_2| = \frac{s_2 \cdot q}{\pi\varepsilon_0 l^2 \cos^2(\gamma/2)} \qquad (1)$$

The charge q will move under influence of the two fixed charges s_1 and s_2 of the measurement apparatus, and finally will arrive at rest at one of the two places where s_1 or s_2 are located.
We propose the following model :
1. We suppose that the place where finally the charge q will end up is determined by the magnitude of the forces of attraction between the three charges. Namely if $|F_1|$ is bigger than $|F_2|$ q will move, and arrive at the place where s_1 is located, and if $|F_1|$ is smaller than $|F_2|$ q will move, and arrive at the place where s_2 is located.

2. The charges s_1 and s_2 are in a certain sense arbitrary. Because at the moment where the charge q is put in the measurement apparatus, we do not know in which state the charging and uncharging



We consider the three charges from fig 1 as they are located in a plane, q is the classical entity located in the point indicated by the direction b, s_1 and s2 of the measuring apparatus are located in the points indicated by the directions a and -a. F1 and F2 are the forces of attraction between s_1 and s and s_2 and s (fig 3).

of the two charges balls, ball 1 and ball 2, of the measuring apparatus is. If this happens in a moment of uncharged charges s_1 and s_2, nothing will happen with the charge q (since there are no forces). The action will only start after the next charging. In any case, since we wait till one of the two outcomes is registered, this experimental situation can be modelled by supposing that for the actual motion of the charge q, and hence for the occurrence of one of the outcomes, s_1 and s_2 are such that s_1 is an at random number in the interval [0,s], and $s_2 = s - s_1$. By means of these two hypothesis 1 and 2, we can make a mathematical derivation for the probabilities P(C, **a, b**) :

$$P(C, \mathbf{a}, \mathbf{b}) = \text{Probability that } |F_1| \text{ is bigger than } |F_2|$$

$$= P\left(\frac{s_1 \cdot q}{\pi\varepsilon_0 l^2 \sin^2(\gamma/2)} > \frac{s_2 \cdot q}{\pi\varepsilon_0 l^2 \cos^2(\gamma/2)} \right)$$

$$= P\left(s_1 \cos^2(\gamma/2) > s_2 \sin^2(\gamma/2)\right)$$

$$= P\left(s_1 \cos^2(\gamma/2) > (s - s_1)\sin^2(\gamma/2)\right) = P\left(s_1 > s \sin^2(\gamma/2)\right)$$

$$= \frac{(s - s \sin^2(\gamma/2))}{s} = \cos^2(\gamma/2). \qquad (2)$$

We see that we find indeed the already experimentally approximated probabilities.

4. The physical meaning of the possibility of constructing a macroscopical physical model with quantum probabilities.

Because these experimental probabilities P(C, **a, b**), through the relative frequencies of repeated measurements, are the **only** connection of a theory with the experiments in our reality, we can describe this classical entity by the spin-formalism of quantum mechanics. Concretely this means that a state of the iron ball of positive charge q in the direction **b** can be represented by a unit vector u_b of a two dimensional complex Hilbert space. If **b** = (cosϕ·sinθ, sinϕ·cosθ, cosθ) where (θ, ϕ) are the spherical coordinates in some fixed coordinate system with origin in the centre of the rigid rod of the measuring apparatus, then we can represent the state of the classical entity by the unit vector

$$u_b = (e^{-i\phi/2} \cdot \cos(\theta/2), e^{i\phi/2} \cdot \sin(\theta/2)) \qquad (3)$$

of a two dimensional complex Hilbert space as is also done for the spin-state of the spin of a spin 1/2 quantum entity. And the measurement m(C, **a**) is represented by means of the self-adjoint operator

$$S_{\alpha\beta} = \begin{pmatrix} \cos\alpha & e^{-i\beta}\cdot\sin\alpha \\ e^{i\beta}\cdot\sin\alpha & -\cos\alpha \end{pmatrix} \qquad (4)$$

where **a** = (cosα·sinβ, sinα·cosβ, cosα). The eigenvalue +1 corresponds to the outcome "**a**-up" and the eigenvalue -1 to the outcome "**a**-down" of the measurement m(C, **a**).The ordinary rules of the quantum mechanical calculations lead to the corresponding probabilities found as limits of the relative frequencies of the repeated measurements m(C, **a**) if the entity is prepared with preparation p(C, **b**).

Also all the other happenings that can be imagined for the case of the spin of the quantum entity of spin 1/2, have their counterpart in our classical model. For example, performing the measurement m(C, **a**) on the classical entity in state u_b prepared by the preparation p(C, **b**) indeed changes this state u_b into another state, depending on the outcome. The state after the measurement is u_a if the outcome "**a**-up" has occurred, and is u_{-a} if the outcome "**a**-down" has occurred. This change of state, that 'really' happens in our classical example, is what often is referred to as the 'collapse of the wave function' in the quantum language . This change of state is governed in our classical example by the action of the Coulomb forces, and hence does not happen instantaneously. In the quantum mechanical situation this 'change of state' should be governed by the interaction between the measurement apparatus MA(Q, **a**), Stern-Gerlach magnet+screen, with the quantum mechanical entity. We do not know the nature of this interaction, and hence cannot speculate about the 'speed' with which it operates.

We have constructed a situation with a macroscopical classical entity that can be described by the quantum formalism in a two dimensional complex Hilbert space. In [4] is indicated how for an arbitrary quantum entity described in an arbitrary complex Hilbert space a similar situation, only containing macroscopical classical entities, can be constructed.

The explanation for the presence of the quantum probabilities that we propose is only a possible explanation. We cannot prove that it is the correct one. We should like to end this section by indicating that our explanation is very plausible from a general physical point of view.

We repeat : We explain the presence of the quantum probabilities, by the fact that there is a 'lack of knowledge' on the state of the measuring apparatus, or on the interaction between the measuring apparatus and the physical entity during the measurement.

Let us show that this explanation is very plausible: If we introduce the probability as approximate relative frequencies of repeated equivalent experiments, we have to be aware that the technique that we use in real laboratory circumstances to decide that we indeed are performing a series of repeated experiments that are 'equivalent' experiments, depends in a very complicated and conventional way (in the sense used by H. Poincaré in [5]) on our prior knowledge of those pieces of reality that we use to construct the experiment, and that we accept to be equivalent. In the two examples that we have given, we have subdivided an experiment in two parts, a preparation of the physical entity, and a measurement. Again for these two parts we can make the same remark. The technique that we use to decide that the preparations and measurements of repeated experiments are equivalent, depend in a very complicated and conventional way on our prior knowledge of those pieces of reality used to construct the preparations and the measurements.

The case where preparations (states) are not really equivalent on a deeper level, has been treated in general by classical probability theory. In this case we say in classical physics that the physical entity is prepared in a 'mixed' state, and not a 'pure' state. The pure states describe the deeper underlying reality for the mixed state. The formalization of this situation leads to a classical statistical theory, making use of classical Kolmogorovian probability models. Examples of this are given in great detail in [6,7].

The case where the measurements are not really equivalent on a deeper level, has not been treated systematically, and cannot be treated by a classical probability theory, once we want to consider several measurements that cannot be executed together. This is exactly the situation that we have artificially created in our macroscopical classical example, and we see that this situation can be described by the quantum mechanical formalism. Indeed we consider repeated measurements m(C, **a**) as equivalent **only** for the fact that the direction of space determined by the rod that connects the two negative charges is the same. But as we have constructed the measurement apparatus (see **C1)**) we can see that on a deeper level, these measurements are not the same. The way in which the charge s will be distributed on the two balls 1 and 2 is in principle different for every measurement. It is this non-equivalence of the measurement on a deeper level that is at the origin of the presence of the probabilities, that we can describe by the quantum mechanical formalism. We could say, m(C, **a**) is a 'mixed' measurement. And the pure measurements would be the ones where we control the distribution of the charge s over the two balls of the measurement apparatus. Now the last question, is this explanation plausible for the situation of a quantum entity ? Indeed, also in this situation, we consider repeated measurements m(Q, **a**) as equivalent **only** because the direction **a** of the constant part of the magnetic field used in the Stern-Gerlach magnet+screen is the same. We do not know anything about the reality of this Stern-Gerlach magnet+screen on deeper levels. Since the Stern-Gerlach magnet+screen is a construction made of an enormous amount of atoms, it is 'sure' that never all these atoms will be found in the same state for a repeated measurement. Hence it is very plausible that there are deeper 'significant' levels, and that m(Q, **a**) is also a 'mixed' measurement. If our explanation for the origin of the presence of the quantum probabilities is correct, the situation in a quantum measurement would be so that we imagine to make always the same measurement m(Q, **a**) in the repeated experiments, but on a deeper level this is not true. And this fact gives rise to the quantum like probabilities, exactly as in the case of the explicit macroscopical classical example.The measuring apparatus MA(C, **a**) that we have introduced for our classical example, can even be adapted as a 'phenomenological' model for the Stern-Gerlach+screen. We remark that along the same lines of thinking, inside the quantum formalism an equation really describing the measurement process and with an interpretation compatible with the one proposed in this paper, has been constructed [8].

This manner of interpreting quantum probabilities would also explain why this probability structure is only found for the description of the entities in the microscopical world. Because indeed, in this microscopical world, we make experiments with macroscopical entities, and it is this fact, namely that the measurement apparatus is macroscopical, while the physical entity is microscopical, that has as a consequence that almost always the 'lack of deeper level' description of the measuring apparatus, introduces these quantum like probabilities. In our macroscopical example we have imitated such a situation. If we accept our explanation, we can conclude by stating that the general structure of a probability model connected with a general physical situation should be quantum like, and it are the classical probability structures that describe the special situations, where we can manipulate the measurement apparatus in such a detailed way, that almost really equivalent measurements can be performed in the repeated experiments, and probability is only introduced from the presence of mixed states. These ideas are explored in more details, by means of more examples in [6,7].

We also want to remark that by using the classical macroscopical entity with a quantum mechanical description put forward in this paper, we can create a classical macroscopical laboratory situation that violates Bell inequalities, exactly with the same numerical values for the expectation values as in the case of the quantum entity of two particles with spins in the singlet spin state. This laboratory situation is explained in detail in [9].

6) References.

1. Aerts, D., " *The construction of reality* ", in preparation.

2. Gerlach,W. and Stern,O.,"*Der experimentelle Nachweis der Richtingsquantelung im Magnetfeld*", Zeitschrift für Physik, **9**,349,(1922).

3. Busch, P. and Schroeck, F.E.Jr., Found. Phys., **19**, 807 (1989).

4. Aerts, D., J. Math. Phys., **27**, 203 (1986) and in " Information Complexity and Control in Quantum Physics ", ed. Blanquiere, A., Diner, S. and Lochak, G., Springer Verlach (1987).

5. Poincaré, H., " La science et l'hypothèse", Flammarion, Paris (1902, 1968).

6. Aerts, D., "*Estudio sobre Modelos de Probabilidad Clásicos y Cuánticos*" In the proceedings of '*la quinta semana científica de la Universidad Nacional Autonoma de Honduras*', (1988).

7. Aerts, D., "*Study of quantum and classical mechanical probability models*" in preparation.

8. Gisin, N. and Piron, C., Letters in Math. Phys. **5**, 379 (1981), and Gisin, N., Helv. Phys. Acta **54**, 457 (1981).

9. Aerts, D., "*A mechanistic classical laboratory situation violating the Bell inequalities with 2·√2, exactly 'in the same way' as its violations by the EPR experiments* ", to be published in Helv. Phys. Acta.