Published as: Aerts, D., 1993, "Quantum Structures due to fluctuations of the measurement situations", Int. J. Theor. Phys., **32**, 2207 - 2220.

Quantum Structures due to fluctuations of the measurement situations.

Aerts Diederik
TENA, Free University of Brussels,
Pleinlaan 2, B-1050 Brussels, Belgium.

Abstract : We want to analyze in this paper the meaning of the non-classical aspects of quantum structures. We proceed by introducing a simple mechanistic macroscopic experimental situation that gives rise to quantum-like structures. We use this situation as a guiding example for our attempts to explain the origin of the non-classical aspects of quantum structures. We see that the quantum probabilities can be introduced as a consequence of the presence of fluctuations on the experimental apparatuses, and show that the full quantum structure can be obtained in this way. We define the classical limit as the physical situation that arises when the fluctuations on the experimental apparatuses disappear. In the limit case we come to a classical structure but in between we find structures that are neither quantum nor classical. In this sense, our approach not only gives an explanation for the non-classical structure of quantum theory, but also makes it possible to define, and study the structure describing the intermediate new situations. By investigating in which way the non-local quantum behavior disappears during the limiting process we can explain the 'apparent' locality of the classical macroscopical world. We come to the conclusion that quantum structures are the ordinary structures of reality, and that our difficulties of becoming aware of this fact are due to pre-scientific prejudices, of which some of them we shall point out.

1. Introduction.

Everybody agrees that quantum theory is very different from classical theories. It is a new mechanics, but also a new probability theory (quantum probability), a new propositional calculus (quantum logic) and a new measurement calculus (* algebra's). Many aspects of these structural differences in these different categories have been investigated and are presented in this conference of the International Quantum Structure Association. During all these years not many results have been obtained concerning an eventual physical explanation for the difference in structure. In our group in Brussels we have been concentrating on this problem, and the main question that we want to consider is the following : "Is it possible to explain from a physical point of view the nature of the quantum structure?". We have been able to derive various results concerning this question and we shall represent them in this paper. In the explanation that we shall put forward the non-classical structures find their origin in two main aspects of physical reality :

1.1 Experiments in general change the states of the entities under consideration.

This fact has often been mentioned in relation with quantum mechanics. Indeed, in the quantum formalism an arbitrary state, represented by a ray of the Hilbert space, is changed by an experiment into another state, which is an eigen-ray of the operator corresponding to the experiment. Classical theories in principle don't give a description of changing states by experiments, although obviously also in the case of most macroscopical entities the states of these entities will be changed by the effect of the experiment. If however this change is deterministic (equivalent experiments on entities in equivalent states provoke the same change), it can easily be incorporated in a classical theory (as we shall see, the basic structures remain classical). Hence this aspect of change of the state by the experiment, although an essential aspect of quantum theory, is not its most characteristic feature, leading to the appearance of the non-classical structures.

1.2 The presence of fluctuations on the experimental-situations resulting in quantum structures

As we know, the change of the state of a quantum entity by an experiment is not a deterministic process. The state changes to an eigenstate of the experiment and with every eigenstate and individual probability of change is connected. This indeterminism has been a great worry for many of the physicists trying to understand quantum physics. In earlier papers (Aerts 1986, 1987, 1992a) we have proposed a possible explanation for the quantum probabilities. The explanation is the following :

Probabilities arrive as limits of relative frequencies of repeated experiments. Repeated experiments mean equivalent experiments performed on equivalent entities in the same states. Classical probabilities arise from the fact that usually one cannot prepare the same states for the equivalent entities, which in technical language means that the prepared states are mixed and not pure. This 'classical' situation gives rise to a classical probability model. Nobody has problems to understand the presence of this kind of classical probabilities, because they originate in a lack of knowledge, that we have, on the real 'pure' state of the prepared entity. The quest for a hidden variable theory, substituting for quantum theory, is in fact an attempt to explain the quantum probabilities in this classical way, as due to a lack of knowledge about the pure states of the prepared entities, these pure states being described by 'hidden variables'. Von Neumann's theorem (Von Neumann 1932) and later refinements (Bell 1966, Gleason 1957, Jauch and Piron 1963, Kochen and Specker 1967, Gudder 1968), but even more the awareness of the fact that such hidden variable theories always lead to classical structures (Boolean propositional calculus, commutative measurement calculus and Kolmogorovian probability), made it seem impossible to attempt to explain the origin of the quantum probabilities in this way. Therefore not so many physicists believed and believe in hidden variable theories. Let us put forward the explanation that we want to propose for the quantum probabilities. Suppose that in the situation of repeated experiments we do succeed in preparing equivalent states (pure states), but it is the equivalence of the experiments that we fail to realize, then also this type of situation must give rise to probabilities. Indeed suppose that we consider two equivalent entities S_1 and S_2 prepared in the same state p, and 'equivalent' experiments e_1 on S_1 and e_2 on S_2. If these experiments e_1 and e_2 are not completely equivalent as to their effect of change on the state p then they will generally lead to different results and different changes of state, although each individual experimental process can be a deterministic process. We have called this a situation of 'hidden measurements' in (Aerts 1986) in analogy with hidden variables. Superficially one could think that such a situation of hidden measurements must also lead to classical structures, because it is essentially a situation of hidden variables of the experimental apparatuses and not of the entity. This is however not true, as we shall show immediately by means of our example. A situation of hidden measurements always leads to a quantum-like probability model, and generates also the other non-classical structures, non-Boolean lattices of properties, non-commutative algebra's of operators, characteristic of quantum theory. It will be one of the aims of this paper to try to understand why this is so. These different hidden measurements, since they are conceived by us as members of macroscopically equivalent experimental situations, are fluctuations on the experimental situation. We shall show that if we introduce in a very natural way a number between 0 and 1 that parameterizes the magnitude of these fluctuations, we can recover the quantum situation for a maximal value 1 of this parameter, and the classical situation for minimal value 0 of the parameter. In between, we find an intermediate situation, giving rise to structures that are neither quantum nor classical, hence probability models that are neither Kolmogorovian nor quantum and sets of properties that are neither Boolean nor quantum. This parameter, presenting the magnitude of the fluctuations on the experimental situation, can describe the limit-process between the micro-world and macro-world. Fluctuations being maximal when experimental apparatuses are macroscopic and entities are microscopical, and fluctuations being minimal when both experimental situations and entities are macroscopic. In section 2 we introduce our example, in its most simple form, as it has been presented in (Aerts 1992b), giving rise to a 'two dimensional' quantummechanical structure. In section 3 we introduce a variation of the fluctuations on the measurement situations, and show that we arrive at structures that are neither quantum nor classical. In section 4 we analyse this procedure for the infinite dimensional case, and find a simple localization effect, that throws a new light on the problem of non-locality.

2. The example.

Some remarks about the general concepts that we shall use. By the state p of a physical entity S at a certain instant t of time, we mean a description of the 'reality' of this physical entity at this instant t of time. Hence when we use the word 'state', we think of the concept of 'pure' state. The so called 'mixed states' we shall regard as probability measures on the set of pure states. The state p can change when time elapses under influence of the outer world, and this change we will call an evolution process. It can also change under influence of an experiment e on the entity, and this change we will call an experimental process.

Let us now introduce our example. We consider a physical entity that is a point particle P that can be, and can move, on the surface of a sphere, with center O and radius r. This particle P is our physical entity (fig 1).

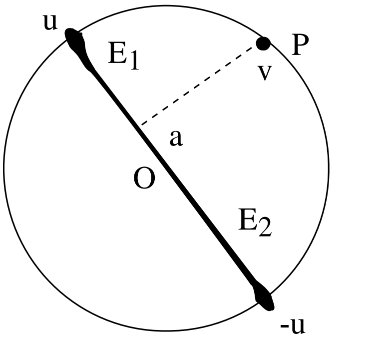

fig 1 : A point particle P is in a state p_v at the point v of the surface of the sphere. The experiment e consists of fixing a piece of elastic with one end in a point u of the surface of the sphere and the other end in the diametrically oposed point -u. Once the elastic is posed, the particle P falls from v onto the elastic and sticks on it in a point a. Then the elastic breaks. Let us consider two parts, the part E_1 from a to u, and the part E_2 from a to -u. If the elastic breaks in E_1, the particle P will be drawn to the point -u, and if it breaks in E_2 it will be drawn to the point u.

In our model of the point particle we consider the point v where the particle is located at a certain instant of time t, as representing the reality of this particle at time t, and hence its state, that we shall denote p_v. We introduce an experiment e_u that is the following. We have a piece of elastic E of length $2r$. This elastic is fixed, with one of its end-points in a point u of the surface of the sphere and the other end-point in the diametrically opposite point $-u$. Hence the elastic passes through the center O of the sphere. Once the elastic is placed, the particle P falls from its original place v onto the elastic, and takes the shortest path when falling, and sticks on it in some point a. Then the piece of elastic breaks. If we consider the two parts of the elastic, the part E_1 from a to u, and the part E_2 from a to $-u$, it must break in a point of one of these two parts. If it breaks in E_2, the particle P will be drawn to the point u by the elastic still connected to it, and we will say that the experiment e_u gives outcome o_1. If it breaks in E_1, the particle P will be drawn to the point $-u$ by the elastic still connected to it, and we will say that the experiment e_u gives outcome o_2. This completes the description of the experiment e_u. If we denote the state of the particle P being in the point u by p_u, and the state of the particle P being in $-u$ by p_{-u}, then we can say that the experiment e_u transforms the state p_v into a new state p_u if outcome o_1 occurs, or a state p_{-u} if outcome o_2 occurs. This change of state is not deterministic, in the sense that the original state p_v can be changed into two different states p_u or p_{-u}. The probabilities connected with either of these two possible changes by the experiment e_u (p_v into p_u, or p_v into p_{-u}) depend on the internal construction of the experimental apparatus, namely the way in which the mechanism of breaking of the elastic functions. We shall make the following hypothesis : The probability that the elastic breaks in a certain segment is proportional to the length of this segment. Under this 'natural' hypothesis we can now easily calculate the probabilities.

We see that the three points v, u and $-u$ are situated in a plane through the diameter of the sphere (see fig 2). Also the point a is in this plane, which means that the point P moves in this plane. Let us call γ the angle between the lines $[0, u]$ and $[0, v]$. Then since $[a, v]$ is orthogonal to $[u, -u]$, and $d(0, u) = r$, $E_1 = d(u, a) = r(1 - cos\gamma) = 2rsin^2\frac{\gamma}{2}$. And $E_2 = d(-u, a) = r(1+cos\gamma) = 2rcos^2\frac{\gamma}{2}$. Since $d(-u, u) = 2r$, we can find the probabilities:

$P(p_u \mid p_v)$ = probability that if P is in v, it will be changed by e_u and end up in u = the probability that E_2 breaks = the length of $[-u, a]$ divided by the length of $[u, -u] = cos^2\frac{\gamma}{2}$

$P(p_{-u} \mid p_v)$ = probability that if P is in v, it will be changed by e_u and end up in $-u$ = the probability that E_1 breaks = the length of $[u, a]$ divided by the length of $[u, -u] = sin^2\frac{\gamma}{2}$

which are the same probabilities as the ones related to the outcomes of a Stern-Gerlach spin experiment on a spin 1/2 quantum particle, of which the spin state in direction $v = (rcos\phi sin\theta, rsin\phi sin\theta, rcos\theta)$ is represented by the vector $(e^{-i\phi/2}cos\theta/2, e^{i\phi/2}sin\theta/2)$ and the experiment corresponding to the spin measurement in direction $u = (rcos\beta sin\alpha, rsin\beta sin\alpha, rcos\alpha)$ by the self adjoint operator $\frac{1}{2}\begin{pmatrix} cos\alpha & e^{-i\beta}sin\alpha \\ e^{i\beta}sin\alpha & -cos\alpha \end{pmatrix}$ in a two dimensional complex Hilbert space, which shows the equivalence between our example with $\epsilon = 1$ and the quantum model of the spin of a spin $\frac{1}{2}$ particle.

It is a well known fact that the probability model corresponding to this physical situation (isomorphic to the probability model of the spin of a spin 1/2 quantum particle) is non-Kolmogorovian. This has been shown in Accardi (1982), and a simple proof can be found in Aerts(1986). Also the lattice of properties of this situation is isomorphic to the lattice of properties of the spin of a spin 1/2 quantum particle, and hence non Boolean, as is explicitly shown in Aerts, Van Bogaert (1992) and Aerts, Durt, Grib, Van Bogaert, Zapatrin (1992). The measurement calculus is non-commutative and also isomorphic to the measurement calculus of the spin of a spin 1/2 particle. In a completely analogous way, models of n-dimensional quantum entities can be constructed, only using macroscopical experimental situations with fluctuations on these experimental situations (Aerts 1986), and a generalization to the infinite dimensional situation has been constructed (Aerts, Durt, Van Bogaert 1993). Let us now proceed by explicitly introducing in a quantitative way the fluctuations on the experimental situations.

3. Introducing a parameterization of the fluctuations on the experimental situations.

If we demand that the elastic can break at every-one of its points, and the breaking of a piece is such that it is proportional to the length of this piece, then this hypothesis fixes the possible fluctuations on the experimental situations. Only certain type of elastic can be used to perform the experiments. We can easily imagine elastic that break in different ways depending on their physical construction. Let us introduce the following different classes of elastic : At the one extreme we consider the elastic that can break in everyone of its points and such that the breaking of a piece is proportional to the length of this piece. These are the ones that we have already considered, and since they lead to a pure quantum structure, let us call them quantum-elastic. At the other extreme, we consider elastic that can only break in one point, and let us suppose, for the sake of simplicity, that this point the middle of the elastic (in Aerts, Coecke, Durt, Van Bogaert 1992, the general situation is treated). This last class are in fact not elastic, but since they are the extreme case of classes of real elastic, we still call them that way. We shall show that if experiments are performed with this class of elastic, the resulting structures are classical, and therefore we will call them classical-elastic. For the general case, we want to consider a class of elastic that can only break in an segment of length $2r \cdot \epsilon$ around the center of the elastic. Let us call these ϵ-elastic. Such an ϵ-elastic of length $2r$ can only break in the points of the interval $[r(1-\epsilon), r(1+\epsilon)]$, and is unbreakable in the points of the intervals $[0, r(1-\epsilon)]$ and $[r(1+\epsilon), 2r]$.

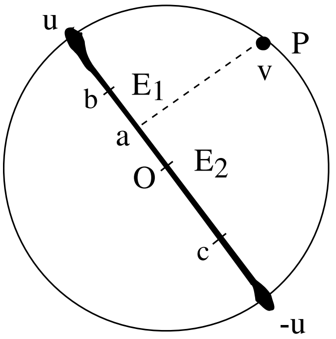

fig 2 : An experiment with an ε-elastic. The elastic can only break in the interval [b,c], where the distance from b to O is $r \cdot \varepsilon$ and the distance from c to O is also $r \cdot \varepsilon$. E_1 is the length of the interval where the elastic can break such that the point P finaly arrives in u (in the dawing this is [b,a]), and E_2 is the length of the interval where the elastic can break such that the point P finaly arrives at -u (in the drawing this is [a,c]).

Clearly the elastic with $\epsilon = 0$, hence the 0-elastic, are the classical-elastic, and the elastic with $\epsilon = 1$, hence the 1-elastic, are the quantum-elastic. In this way, the parameter ϵ can be interpreted as representing the magnitude of the fluctuations present in the experimental situations. If $\epsilon = 0$, and for the experiment e_u only among classical-elastic is chosen, there are no fluctuations, in the sense that all elastic will break in the same point and have the same influence on the changing of the state of the entity. The experiment e_u is then a pure experiment. If $\epsilon = 1$, and for the experiment e_u only among the quantum-elastic is chosen, the fluctuations are maximal, because the chosen elastic can break in any of its points. In fig 2 we have represented a typical situation of an experiment with an ϵ-elastic, where the elastic can only break between the points b and c. Let us calculate the probabilities $P(p_u \mid p_v)$ and $P(p_{-u} \mid p_v)$ for state-transitions from the state p_v of the particle P before the experiment e_u to one of the states p_u or p_{-u}. The length of the interval $[b, c]$ is equal to $r(1 + \epsilon) - r(1 - \epsilon) = 2r \cdot \epsilon$. Different cases are possible : If the point a lies between c and $-u$, then $E_2 = 0$. If the point a lies between b and c (see fig 3), then $E_2 = r(1 + cos\gamma) - r(1 - \epsilon) = r(\epsilon + cos\gamma)$. And if the point a lies between u and b then $E_2 = 2r \cdot \epsilon$. To find a general expression for the probabilities, we write E_2 as a function of γ. To do this we proceed as follows. Let us introduce an angle λ such that $cos\lambda = \epsilon$, and characteristic functions of intervals $X[\alpha, \beta](\gamma) = 1$ for γ belonging to the interval $[\alpha, \beta]$, while $X[\alpha, \beta](\gamma) = 0$ for γ not belonging to the interval $[\alpha, \beta]$. Then $E_2(\gamma) = 2r \cdot \epsilon \cdot X_{[0,\lambda[} + r(cos\lambda + cos\delta) \cdot X_{[\lambda,\lambda+\frac{\pi}{2}]} + 0 \cdot X_{]\lambda+\frac{\pi}{2},\pi]}$. We can now easily calculate the probability $P_\lambda(p_u \mid p_v)$ that the particle P will arrive at u. This is E_2 divided by $2r \cdot e\epsilon$. Hence $P_\lambda(p_u \mid p_v) = \frac{1}{2r\epsilon} \cdot (2r \cdot \epsilon X_{[0,\lambda[} + r(cos\lambda + cos\gamma) \cdot X_{[\lambda,\lambda+\frac{\pi}{2}]} + 0 \cdot X_{]\lambda+\frac{\pi}{2},\pi]}) = 1 \cdot X_{[0,\lambda[} + \frac{1}{2cos\lambda} \cdot (cos\lambda + cos\gamma) \cdot X_{[\lambda,\lambda+\frac{\pi}{2}]} + 0 \cdot X_{]\lambda+\frac{\pi}{2},\pi]}$ An analogous calculation gives us $P(p_{-u} \mid p_v) = 0 \cdot X_{[0,\lambda[} + \frac{1}{2cos\lambda} \cdot (cos\lambda - cos\gamma) \cdot X_{[\lambda,\lambda+\frac{\pi}{2}]} + 1 \cdot X_{]\lambda+\frac{\pi}{2},\pi]}$. So we have :

$$P_\lambda(p_u \mid p_v) = 1 \cdot X_{[0,\lambda[} + \frac{1}{2cos\lambda} \cdot (cos\lambda + cos\gamma) \cdot X_{[\lambda,\lambda+\frac{\pi}{2}]} \tag{1}$$

$$P(p_{-u} \mid p_v) = \frac{1}{2cos\lambda} \cdot (cos\lambda - cos\gamma) \cdot X_{[\lambda,\lambda+\frac{\pi}{2}]} + 1 \cdot X_{]\lambda+\frac{\pi}{2},\pi]} \tag{2}$$

Let us verify whether the classical case ($\epsilon = 0$, hence $\lambda = \frac{\pi}{2}$), and the quantum case ($\epsilon = 1$, hence $\lambda = 0$) arrive as limits of the general case.

3.1. The classical limit ($\epsilon \to 0$, and $\lambda \to \frac{\pi}{2}$):

a) The state p belongs to the Northern hemisphere (γ belongs to the interval $[0, \frac{\pi}{2}[$).

Then if $\epsilon \to 0$, or equivalently $\lambda \to \frac{\pi}{2}$, there is a moment that λ is bigger than γ, then we have $P_\gamma(p_u \mid p_v) = 1$ and $P_\gamma(p_{-u} \mid p_v) = 0$. The points v of the Northern hemisphere all arrive at u.

b) The state p belongs to the Southern hemisphere (γ belongs to the interval $]\frac{\pi}{2}, \pi]$).

Then if $\epsilon \to 0$, or equivalently $\lambda \to \frac{\pi}{2}$, there is a moment that λ is smaller than γ, then we have $P_\lambda(p_u \mid p_v) = 0$ and $P_\lambda(p_{-u} \mid p_v) = 1$. The points v of the Southern hemisphere all arrive at $-u$.

c) The state p belongs to the equator ($\gamma = \frac{\pi}{2}$). Then $P_\lambda(p_u \mid p_v) = \frac{1}{2cos\lambda} \cdot (cos\lambda) = 1/2$, and $P_\lambda(p_{-u} |p_v) = \frac{1}{2cos\lambda} \cdot (cos\lambda) = 1/2$. The points of the equator have probability 1/2 to arrive at u, and probability 1/2 to arrive at $-u$. This corresponds to the classical unstable equilibrium situation, a classical indeterminism.

3.2 The quantum limit ($\epsilon \to 1$, and $\lambda \to 0$) :

Then $P_\lambda(p_u \mid p_v) = 1/2(1 + cos\gamma) = cos^2(\frac{\gamma}{2})$ and $P_\lambda(p_{-u} \mid p_v) = 1/2(1 - cos\gamma) = sin^2(\frac{\gamma}{2})$.

3.3 An intermediate case, $\epsilon = 1/2$, and $\lambda = \frac{\pi}{3}$.

Let us calculate explicitly the probabilities for this case (see fig 4) :

a) For γ smaller than $\frac{\pi}{3}$: Then $P_\lambda(p_u \mid p_v) = 1$, and $P_\lambda(p_{-u} \mid p_v) = 0$. This is a zone, of the form of a spherical sector, with eigen-states of the experiment e, with eigen-outcome o_1. All the states p_v of this zone will arrive in p_u.

b) For γ bigger than $2\frac{\pi}{3}$: Then $P_\lambda(P_u \mid p_v) = 0$, and $P_\lambda(p_{-u} \mid p_v) = 1$. This is a zone, of the form of a spherical sector, with eigen-states of the experiment e, with eigen-outcome o_2. All the states of this zone will arrive in $-u$.

c) For γ between $\frac{\pi}{3}$ and $2\frac{\pi}{3}$: All the states in this region are superposition states. Let us calculate some probabilities :
$\gamma = \frac{\pi}{3}$, then $P_\lambda(p_u \mid p_v) = 1$, and $P_\lambda(p_{-u} \mid p_v) = 0$.
$\gamma = \frac{\pi}{2}$, then $P_\lambda(p_u \mid p_v) = 1/2$, and $P_\lambda(p_{-u} \mid p_v) = 1/2$.
$\gamma = 2\frac{\pi}{3}$, then $P_\lambda(p_u \mid p_v) = 0$, and $P_\lambda(p_{-u} \mid p_v) = 1$.

This shows that the earlier quantum-region is now concentrated here, in the zone where γ is between $\frac{\pi}{3}$ and $2\frac{\pi}{3}$.

This example shows very well the effect of the fluctuations on the experimental situations. We have to ask ourselves now whether it would be possible to perform experiments to be able to verify whether these models correspond in one way or another to the physical reality. We should look for experiments in the field of mesoscopic physics. Can an ϵ-elastic model describe the spin 1/2 behavior of a huge spin 1/2 molecule? If this would be the case, then our theory about the classification with decreasing fluctuations in the experimental situations would be able to clarify the mystery of the micro-world going over in the macro-world. In the next section we explain a localization model constructed in the same way.

4. A localization procedure.

The ϵ-example in two dimensions, as it has been presented in the foregoing chapters, gives us a clear insight the classical limit process. Because of the limiting number of dimensions, it is not clear what this process becomes for an arbitrary quantum mechanical entity, of which the state is described by a wave function $\psi(x)$ element of $L^2(\Re^3)$. We have studied the procedure in the n-dimensional situation, and considered the limit for $n \to \infty$, which brings us in the situation of a general quantum-entity, and a marvelous simple procedure results. For those who want to read the details of the construction, we refer to (Aerts, Durt, Van Bogaert 1993), we only present the result here. We investigate the situation where we have an experiment e that is represented by a self-adjoint operator A_e, and the spectrum of this self adjoint operator is

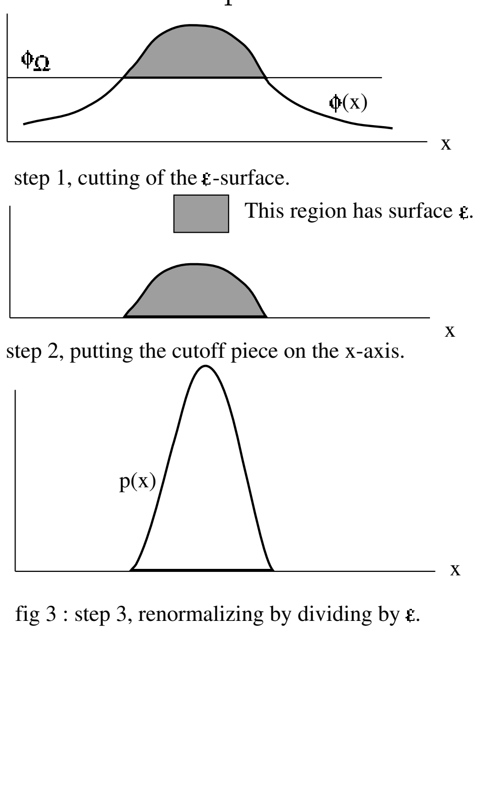

fig 3 : step 3, renormalizing by dividing by ε.

a subset of the set $\Re$ of real numbers. The state p of the entity S is now represented by a complex function $\psi(x)$ element of $L^2(\Re^3)$. We write the wave-function $\psi(x) = \rho(x)e^{iS(x)}$, where $\rho(x)$ is a positive function, and then we know that $\phi(x) = \rho^2(x)$ represents the probability amplitude of the wave function $\psi(x)$. We have an ϵ given, and find the following procedure. We cut, by means of a constant function ϕ_Ω, a piece of the function $\phi(x)$, such that the surface contained in the cutoff piece equals ϵ (see step 1 of fig 3). We move this piece of function to the x-axis (see step 2 of fig 3). And then we renormalize by dividing by ϵ (see step 3 of fig 3), and this gives us a function $\phi^\epsilon(x)$. We define a new wave-function $\psi^\epsilon(x) = \sqrt{\phi^\epsilon(x)}e^{iS(x)}$. If we proceed in this way for smaller values of ϵ, we shall finally arrive at a delta-function for the classical limit $\epsilon \to 0$, and the delta-function is located in the original maximum of the quantum probability distribution. For $\epsilon = 1$ we find the original wave-function $\psi(x)$.

Many aspects of the relation between quantum mechanics and classical mechanics can be investigated using this classical limit procedure. We only want to mention one, the problem of non-locality. Let us investigate what becomes of the non-local behavior of quantum entities taking into account the classical limit procedure that we propose in this paper. Suppose that we consider a double slit experiment, then the state p of a quantum entity having passed the slits can be represented by a probability function $p(x)$ of the form represented in fig 4. We can see that the non-locality presented by this probability function gradually disappears when ϵ becomes smaller, and in the case where $p(x)$ has only one maximum finally disappears completely. When there are no fluctuations on the measuring apparatus used to detect the particle, it shall be detected with certainty in one of the slits, and always in the same one. If $p(x)$ has two maxima (one behind slit 1, and the other behind slit 2) that are equal, the non-locality does not disappear. Indeed, in this case the limit-function is the sum of two delta-functions (one behind slit 1 and one behind slit 2). So in this case the non-locality remains present even in the classical limit. If our procedure for the classical limit is a correct one, also macroscopical classical entities can be in non-local states. How does it come that we don't find any sign of this non-locality in the classical macroscopical world? This is due to the fact that the set of states, representing a situation where the probability function has more than one maximum, has measure zero, compared to the set of all possible states, and moreover these state are 'unstable'. The slightest perturbation will destroy the symmetry of the different maxima, and hence shall give rise to one point of localization in the classical limit. Also classical macroscopical reality is non-local, but the local model that we use to describe it gives the same statistical results, and hence cannot be distinguished from the non-local model.

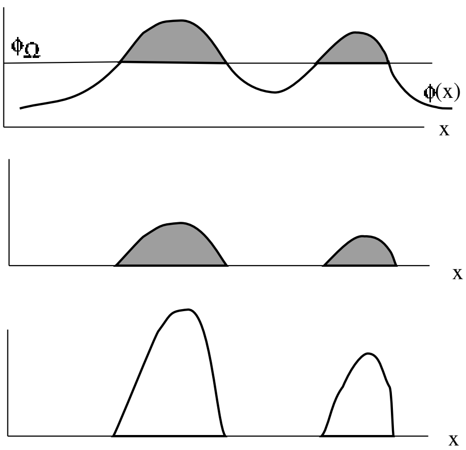

fig 4 : the classical limit procedure in the situation of a non-local quantum state.

We also want to remark that all the interference phenomena remain while taking the classical limit, since the phase factor $exp(iS(x))$ of the wave function $\psi(x)$ is not changed. But the places where these interference effects can be detected are restrained more and more if $\epsilon \to 0$, till they finally are only located in the original maximum of the amplitude. We could say that the quantum interference phenomena localize as well, when the fluctuations decrease with a decreasing ϵ.

5. Conclusion.

The approach that we present here, although it still has to be developed in many aspects, provides an answer to the question that we have pointed out in the introduction. The existence of fluctuations of internal variables of the experimental apparatuses, their magnitude labeled by a parameter ϵ, gives rise to quantum-like structures in all the categories that we have pointed out. It generates non-Boolean lattices of properties (not explicitly shown in this paper, but we refer to Aert, Coecke, Durt, Van Bogaert 1992 for a detailed presentation of the example and its lattice of properties), it generates non-Kolmogorovian probability models (see Aerts 1986), and it also gives rise to non-commutative structures of observables (a detailed exposition in this category is prepared). As we mentioned already, these fluctuations on the experimental apparatuses can be interpreted as 'hidden-variables', but then they are highly contextual, since each experiment brings about a different set of hidden-variables. So they are not 'hidden variables' of a 'classical hidden variable theory', because they do not deliver an 'additional deeper' description of the reality of the physical entity. Their presence, as variables of the experimental apparatuses, has a well defined philosophical meaning, and expresses that we, human beings, want to construct a model of reality, independent of the fact that we experience this reality. The reason is that we look for 'properties' or 'relations of properties', and they are defined by our ability to make predictions independent of experience. We want to model the structure of the world, independent of us observing and experimenting with this world. Since we don't control these variables in the experimental apparatuses, we shall not allow them in our model of reality, and the probability introduced by them cannot be eliminated from a predictive theoretical model. In the macroscopical world, because of the availability of many experiments with neglecting fluctuations, we find an 'almost' deterministic model. Indeed remember that in the classical limit, the classical type of indeterminism, that we all know very well to exist, remains. In other regions of reality, where these kind of experiments are not available, the model shall be non-classically indeterministic. The pre-material world is such a region. The explicit classical limit for an arbitrary quantum entity in our approach explains why non-local states, and also quantum-interference become un-detectable in the classical world. Philosophically speaking however non-locality, and quantum-interference, do not disappear, and are a fundamental property of nature (see in relation with this question also Aerts, Reignier 1991). From this follows that we have to belief that our model of space, as the theater in which all entities are present and move around, should be considered partly as a human construction, due to our human experience with the macroscopical material entities. Space is the structure that can contain all entities that we know by means of properties for which we have measurements with neglecting fluctuations at our disposal. Entities that do not allow us to characterize them only by means of neglecting-fluctuations-measurements cannot be fitted into space. They have no place, and can only be detected, which means that we have forced them in a state, where we can measure their position with a neglecting-fluctuation-measurement. This finding shall force us to review completely the concept of space and its relation with reality.

6. References.

Accardi, L. and Fedullo, A. (1982), Lett. Nuovo Cimento, **34**, 161.

Aerts, D. (1986). *"A possible explanation for the probabilities of quantum mechanics"*, J. Math. Phys. **27**, 203.

Aerts, D. (1987). *"The origin of the non-classical character of the quantum probability model"*, in *"Information, Complexity and Control in Quantum Physics"*, ed. Blanquiere, A., et al., Springer Verlach.

Aerts, D. (1992a), *"A macroscopical classical laboratory situation with only macroscopical classical entities giving rise to a quantum mechanical description"*, in *"Quantum Probabilities, volume VI"* ed., Accardi, L., World Scientific.

Aerts, D. (1992b), *In search of the meaning of quantum structures*, proceedings of the ANPA meeting, Stanford University (1992)

Aerts, D. and Van Bogaert, B. (1992). *"A mechanical classical laboratory situation with a quantum logic structure"*, International Journal of Theoretical Physics,**10** ,1839.

Aerts, D., Durt, T., Grib, A.A., Van Bogaert, B. and Zapatrin, R.R. (1992). *"Quantum Structures in Macroscopical Reality"*, International Journal of Theoretical Physics, **32** , 489.

Aerts, D., Durt, T. and Van Bogaert, B. (1992). *"A physical example of quantum fuzzy sets and the classical limit"*, in the proceedings of the International Conference on Fuzzy Sets, Liptovsky.

Aerts, D., Coecke, B., Durt, T. and Van Bogaert, B. (1992), *"Quantum, Classical and Intermediate ."*, to be published in the proceedings of the third winter school on measure theory, Liptovsky, 1993.

Aerts, D., Durt, T. and Van Bogaert, B. (1993), *"Quantum Indeterminism, the Classical Limit and Non-Locality"*, proceedings of the Symposium on the Foundations of Modern Physics, Helsinki, World Scientific Publishing Company (1993).

Aerts, D. and Reignier, J., (1991). *"On the problem of non-locality in quantum mechanics"*, Helv. Phys. Acta, **64**, 527.