THE HIDDEN-MEASUREMENT FORMALISM:
QUANTUM MECHANICS AS A CONSEQUENCE
OF FLUCTUATIONS ON THE MEASUREMENT

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We expose a formalism, that we have called the 'hidden measurement formalism', where the quantum structure is due to the presence of 'fluctuations' on the interaction between the system and the measurement apparatus. In this formalism the quantum mechanical probabilities are not ontological but arise as a consequence of 'lack of knowledge' about this interaction. We study the quantum classical limit and the EPR problem in the light of this explanation.

Key words: hidden measurements, determinism, paradoxes.

1. HIDDEN MEASUREMENTS

To understand quantum mechanics many have sought to supplement the quantum state with additional variables that would allow a more complete description of the system. One problem associated with these so-called hidden variables is that, once we supplement the state of the entity with additional variables, one restores the Kolmogorovian character of the probabilities. In this case a measurement is an observation of an entity and the probabilistic character arises because there is a lack of knowledge about the precise state of the entity. In a sense this explanation is 'too classical' as is demonstrated by the many no-go theorems [11]. Bohr stressed the importance of taking into account the whole experimental setup because the setup would determine in an essential way the possible set of expected outcomes. We want to push this idea into rigorous mathematics and develop a theoretical framework wherein the probabilities of quantum mechanics arise not because of the indefinite character of the state of the entity, but because of the indefinite state of the measurement equipment and the lack of knowledge about the interaction between system and measurement apparatus. Such an
approach has been elaborated [1,2,3] by the Brussels group under the following two hypotheses: (1) a measurement induces a 'real' state transition (2) the probability related to an outcome of an experiment is due to a lack of knowledge about the interaction between the measurement apparatus and the entity under observation. Let us first introduce a model that explicitly incorporates these features and constitutes a model of a two-dimensional Hilbert space quantum entity.

2. THE QUANTUM MACHINE

The machine (see also [1,2,3]) that we consider consists of a physical entity $S$ that is a point particle $P$ that can move on the surface of a unit sphere, denoted $\text{surf}$. The unit-vector $v$ where the particle is located on $\text{surf}$ represents the state $p_v$ of the particle (see Fig. 1(a)). For each point $u \in \text{surf}$, we introduce the following experiment $e_u$. We consider the diametrically opposite point $-u$, and install a piece of elastic of length 2, such that it is fixed with one of its end-points in $u$ and the other end-point in $-u$. Once the elastic is installed, the particle $P$ falls from its original place $v$ orthogonally onto the elastic, and sticks on it (Fig. 1(b)). Then the elastic breaks and the particle $P$, attached to one of the two pieces of the elastic (Fig. 1(c)), moves to one of the two end-points $u$ or $-u$ (Fig. 1(d)). Depending on whether the particle $P$ arrives in $u$ (as in Fig. 1) or in $-u$, we give the outcome $o_1^u$ or $o_2^u$ to $e_u$. We can easily calculate the probabilities corresponding to the two possible outcomes. Therefore we remark that the particle $P$ arrives in $u$ when the elastic breaks in a point of the interval $L_1$ (which is the length of the piece of the elastic between $-u$ and the point where the particle has arrived, or $1 + \cos \theta$), and arrives in $-u$ when it breaks in a point of the interval $L_2$ ($L_2 = L - L_1 = 2 - L_1$). We make the hypothesis that the elastic breaks uniformly, which means that the probability that the particle, being in state $p_u$, arrives in $u$, is given by the length of $L_1$ divided by the length of the total elastic (which is 2). The probability that the particle in state $p_v$ arrives in $-u$ is the length of $L_2$ (which is $1 - \cos \theta$) divided by the length of the total elastic. If we denote these probabilities respectively by $P(o_1^u, p_u)$ and $P(o_2^u, p_u)$ we have:

$$P(o_1^u, p_u) = \frac{1 + \cos \theta}{2} = \cos^2 \frac{\theta}{2},$$

(1)

$$P(o_2^u, p_u) = \frac{1 - \cos \theta}{2} = \sin^2 \frac{\theta}{2},$$

(2)

These transition probabilities are the same as the ones related to the outcomes of a Stern-Gerlach spin experiment on a spin $-1/2$ quantum particle, of which the quantum spin-state in direction $\nu = (\cos \phi \sin \theta,$
Fig. 1. A representation of the quantum machine. In (a) the physical entity $P$ is in state $p_v$ in the point $v$, and the elastic corresponding to the experiment $e_u$ is installed between the two diametrically opposed points $u$ and $-u$. In (b) the particle $P$ falls orthogonally onto the elastic and sticks to it. In (c) the elastic breaks and the particle $P$ is pulled towards the point $u$, such that (d) it arrives at the point $u$, and the experiment $e_u$ gets the outcome $0^u$.

$\sin \phi \sin \theta, \cos \theta$, denoted by $\psi_u$, and the experiment $e_u$ corresponding to the spin experiment in direction $u = (\cos \beta \sin \alpha, \sin \beta \sin \alpha, \cos \alpha)$, is described respectively by the vector and the self adjoint operator

$$\psi_u = (e^{-i\phi/2} \cos \theta/2, e^{i\phi/2} \sin \theta/2),$$  \hspace{1cm} (3.a)

$$H_u = \frac{1}{2} \left( \begin{array}{ccc} \cos \alpha & e^{-i\beta} \sin \alpha \\ e^{i\beta} \sin \alpha & \cos \alpha \end{array} \right)$$  \hspace{1cm} (3.b)

of a two-dimensional complex Hilbert space.

3. QUANTUM-CLASSICAL, NON LOCALITY AND EPR

A most interesting consequence of viewing quantum probabilities as being due to fluctuations of the interaction between the measurement and the entity that is being measured, is that one can easily parameterize these fluctuations. This was done [7] with a parameter called epsilon and the model was baptized the epsilon-model (see also [5]). One can study the behaviour of the epsilon model for varying epsilon values, and we have done
this within three different frameworks: the lattice approach, the probabilistic approach and the *-algebra approach to quantum mechanics.

3.a. The Lattice Approach

For intermediate values of $\epsilon$, that is $0 < \epsilon < 1$, we find that 2 of the 5 axioms needed in the lattice approach to reconstruct quantum mechanics are violated. The axioms that are violated are the weak-modularity and the covering law, and it are precisely those axioms that are needed to recover the vector space structure of the state space in quantum mechanics (see [7] for details).

3.b. The Probabilistic Approach

If we take the case of vanishing fluctuations, do we obtain the Kolmogorovian theory of probability? This would be most interesting, since then we would have constructed a macroscopic model with an understandable structure (i.e., we can see how the probabilities arise) and a quantum and a classical behaviour. One of the authors [3] proposed to test the polytopes for a family of conditional probabilities. The calculations can be found in [9] and the result was what we hoped for: a macroscopic model with a quantum and a Kolmogorovian limit (see also [12] for a detailed analysis of the quantum-classical relation in this framework).

3.c. The *-Algebra Approach

The *-algebra provides a natural mathematical language for quantum mechanical operators. We applied the concepts of this approach to the epsilon model to find that an operator corresponding to an $\epsilon$-measurement is linear if and only if $\epsilon = 1$ [10].

Using the sphere model, we can illustrate the issue of non-locality. Picture the sphere model and a quantum entity located somewhere on the sphere. Metaphorically, we visualise the space context and the momentum context as orthogonal observables, thus as orthogonal directions for the piece of elastic. A measurement of the position of the entity will give a definite outcome, dragging the entity into one of the possible spatial outcomes, that is, dragging the entity into the space context. Similarly, a measurement of the momentum observable will drag that same entity along an orthogonal direction into one of the eigenstates of the momentum operator. This means that the locus and the momentum are partly created during the measurement. The state of the entity before the measurement has no definite momentum or locus (in the ontological sense), but is nevertheless definite
itself, being a pure state. We arrive at a conclusion that is very difficult for us to visualise: Space is not the theatre of reality, but a context. The same applies for momentum (see [4] for more details).

In [8] we concentrated on the question as to the nature of the classical limit procedure of a general quantum entity. The result gives one an easy way of picturing the classical limit. Given a probability distribution \( \rho(x) \) in the case of zero fluctuations, we obtain the distribution \( \rho_\epsilon(x) \), for the case of \( \epsilon \) fluctuations, by the following procedure. By means of a constant function we cut off a piece of the original distribution \( \rho(x) \), such that the surface contained in the cutoff piece equals \( \epsilon \). Next we move this cut function down to the x-axis and renormalise by dividing with \( \epsilon \). If we proceed this way for ever decreasing values of \( \epsilon \), we finally obtain a Dirac distribution for the classical limit in the case we have only one maximum, with the peak of the distribution located at the original maximum.

In 1996 [6] we focused on the question of the meaning of the Bell-inequality. A model [2] for singlet experiments like Aspect's based on the hidden measurements that reproduces the predictions of quantum mechanics for all possible angles of the polarizer was constructed in 1991. The "non-local" element was introduced explicitly by means of a rod that connected two sphere-models. We parametrized the correlation that the rod provided. The result of this construction is that one violates the Bell-inequalities even more for classical but non-locally connected systems, that is, \( \epsilon = 0 \), illustrating that the violation of the Bell-inequality is due to the non-locality rather then to the indeterministic character of quantum theory.

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5. REFERENCES