A MODEL WITH VARYING FLUCTUATIONS IN THE MEASUREMENT CONTEXT

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In the "hidden measurement formalism," we try to explain the emergence of the quantum probabilities from the presence of fluctuations in the measurement context. We use a model that was constructed by Aerts et al. as a metaphorical model to extend these ideas.

Key words: hidden measurements.

1. A CLASSICAL SPIN-1/2 MODEL

In a paper that was published in 1986 [1], Aerts introduced a hidden variable model where the quantum probabilities arise because of a lack of knowledge on the experimental context during the measurement process. One version of this model consists of a particle, corresponding to the physical entity, moving on the surface of the two-dimensional unit sphere S^2; the experimental setup corresponds to the choice of two opposite points on the sphere. The interaction during the measurement consists in the following: the particle is orthogonally projected on the connecting line between these two points. To visualise the model, we assume the connecting line to be a sticky elastic that will break at a random place after contact with the particle, and pulls it to one of the endpoints (Fig. 1). If we know both the initial state and the breaking point of the elastic, we can predict the result of the measurement with absolute certainty. This model is isomorphic to the case of a spin-1/2 particle in a Stern-Gerlach experimental device. Since the hidden parameter belongs to the global experimental set-up, this idea was aptly dubbed the "hidden measurement" approach. One can generalise and prove that it is always possible to find such a hidden measurement explanation for general non-classical probability structures [2,6,7].

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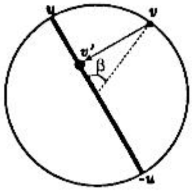

Fig. 1. Illustration of a measurement **u** on an Aerts' spin-$\frac{1}{2}$ entity when the initial state is **v**.

For this model we can easily prove our claim in [8]. If we consider a single measurement in the **u**-direction, we can introduce a parameter $\lambda, -1 \leq \lambda \leq 1$ that indicates the place of rupture on the measurement elastic. It is easy to see that we can use the same set $\Lambda = [-1, +1]$ for all measurements, in all possible directions. We shall consider Jauch and Piron's no-go theorem [9] and take a closer look at the dispersion-free components $(\mathbf{p}, \lambda)$ in our model. If we perform an experiment in the **u**-direction we find:

$$(\mathbf{p}, \lambda)(\mathbf{u}) = \begin{cases} 0 & \lambda > \cos(\beta), \\ 1 & \lambda \leq \cos(\beta), \end{cases}$$

with β the angle between **p** and **u**. If we take two experiments in different directions **u** and **v** and an initial state **p** and choose $\lambda < min\{\cos(\beta_1), \cos(\beta_2)\}$, we see that $(\mathbf{p}, \lambda)(\mathbf{u}) = (\mathbf{p}, \lambda)(\mathbf{v}) = 1$, while $(\mathbf{p}, \lambda)(\mathbf{u} \wedge \mathbf{v}) = (\mathbf{p}, \lambda)(0) = 0$. Therefore the dispersion-free components do not satisfy the Jauch-Piron condition, and we can not apply the theorem.

2. GENERALISATION OF THE MODEL

In a generalisation of this model, constructed by Aerts et al. [3,4,5], one considers the lack of knowledge on the measurement context to disappear in a continuous way. Using the metaphorical elastic of our earlier example, one can easily accomplish this by introducing a parameter $\epsilon, 0 \leq \epsilon \leq 1$, describing in a simple but quantitative way the magnitude of the fluctuations, with 2ϵ the length of the fragile domain on this string and **-u** as lower bound for an experiment in the **u**-direction. For fixed ϵ, the resulting model is a particular case of the so-called ϵ-model. It is easy to compute the transition probabilities if we perform such a measurement in the **u**-direction, for an entity in a state **v**:

$$\mathcal{P}[\mathbf{u}|\mathbf{v}] = 1 \; - \; \mathcal{P}[-\mathbf{u}|\mathbf{v}] = 1 \quad (2\epsilon - 1 < \mathbf{u} \cdot \mathbf{v}),$$

$$\mathcal{P}[\mathbf{u}|\mathbf{v}] = 1 - \mathcal{P}[-\mathbf{u}|\mathbf{v}] = \frac{1 + \mathbf{u} \cdot \mathbf{v}}{2\epsilon} \quad (2\epsilon - 1 \geq \mathbf{u} \cdot \mathbf{v}).$$

If $\epsilon = 1$, we can represent the propositional structure of the model in the two-dimensional Hilbert space $\mathcal{C}^2$. Varying ϵ, the model yields a structural transition between quantum behaviour on the one hand ($\epsilon = 1$) and a classical structure on the other ($\epsilon = 0$), yielding purely deterministic experiments in the latter case. Indeed, only in these two limit cases the propositional lattice generated by the model satisfies all five of Piron's axioms necessary to apply his well-known representation theorem [10]. For intermediate ϵ, the lattice is not weakly modular and it doesn't satisfy the covering law [3]. Both these axioms are needed to impose a vector space structure on the representation space of the entity.

In a following step, we can slightly generalise this model by permitting fluctuations of varying magnitude in the measurement context, but always letting $\epsilon > 0$. In this case, the classical situation would only appear if we are able to accurately control the measurement environment, corresponding to the possibility of choosing an experiment with an arbitrary small value of ϵ among the set of all possible experiments. It can be shown that in this case one generates the standard topology on S^2, while the strictly classical case would correspond with the discrete topology [5].

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