REDUCTION*

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To avoid misunderstanding, it is desirable to specify here a meaning of reduction which we shall *not* consider in the following. 'Reduction' has been used (especially by logical positivists) to denote the claim that a scientific theory can be expressed entirely in terms of observation statements, or, weaker, that all theoretical terms of a theory can be so expressed. We make no such claims and shall understand by 'reduction' the following three types.

(1) Logical reduction

(2) Theory reduction (sometimes called 'semantic reduction')

(3) Reductive explanation (sometimes called 'explanatory reduction')

1 Logical reduction

Given a theoretical structure, S, logical reduction is the process in which it is proven that this structure is replaceable by a set of fundamental propositions, T, such that S becomes a logical consequence of T. The axiomatic method of describing a theory can be called the result of a 'logical reduction'. In its

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purest form, this occurs in mathematics. Hilbert's axiomatization of Euclidean geometry is a beautiful example (Hilbert 1902). Similarly, the uniqueness of the natural numbers (up to isomorphisms) can be considered to be the result of a logical reduction (Wilder 1952, Ch.VI): they can be expressed as a Peano system and thereby reduced to the axioms of set theory; their uniqueness then follows too. This process is completely devoid of contingencies of nature, a crucial element of all sciences. It applies therefore only to mathematical structures or to the mathematical components of a scientific theory.

The axiomatic method has its serious difficulties. First, after proving the consistency of the axioms and possibly also their independence, there is the problem of completeness which, since Gödel is well-known to be insoluble (Gödel 1931). Second, there is the need of having in the foundations of the theoretical structure in addition to the axioms, a set of *undefined terms* (terms like 'point' and 'line' in the case of geometry). And third, there is the *logic* by which the theoretical structure is deduced from the axioms: why does it have to be Aristotelian logic? Many other logics are possible. For example, in the case of quantum mechanics, an attempt at axiomatization led to the realization that it cannot be done by means of Aristotelian logic (a Boolean lattice). One can prove that the distributive law cannot be part of it and that a non-Boolean lattice of propositions must be chosen (Jauch 1968). This choice is not unique.

A very special case of logical reduction is the equivalence of two structures in the sense that both are equivalent representations of the same mathematical object. For example, S and T can be equivalent if they both are faithful representations of a group.

A classic example of such an equivalence occurred in the history of physics when the Schrödinger and the Heisenberg theories of quantum mechanics (originally proposed as quite different theories), were shown to be equivalent representations of the von Neumann formulation of quantum mechanics in terms of a Hilbert space.

Logical reduction is a formal procedure that can be used in a scientific theory only *post facto*, after the theory has been formulated based on empirical information. This does not prevent a theorist from postulating a scientific law in the process of constructing a theory. But in no known case does axiomatization of a theory help to elucidate the scientific problems one encounters.

2 Theory Reduction

By 'theory' is here meant a scientific theory. Such a theory is a complex consisting of various components including a set of central terms (vocabulary) with defined meaning (semantics), a set of fundamental laws, often a mathematical structure (especially in the physical sciences), a domain of validity (range of applicability), and an ontology. The theory implies *laws* which can be compared with empirical evidence. To this effect, a *model* is constructed which (in a suitably idealized way) represents the phenomenon that is to be observed.

A theory S is said to be reduced to a theory T if and only if T 'implies' S. The difficulties of theory reduction lie in the explanation of the notion of 'implication' as used here (Nagel 1961). Clearly, it is not a purely logical implication as in logical reduction because S and T differ not only in their mathematical structure but also in vocabulary and semantics. There are terms in S that have no meaning in T and *vice versa*. Such vocabularies are sometimes called 'incommensurable' (Kuhn 1962).

It is very important that 'theory reduction' as used here refers to theories on *different levels of reality*. This is the reason for the existence of incommensurable terms: different levels involve different ontologies. The relationship between theories on the same level do not cause difficulties but are typically related by inclusion: S is a sub-theory of T. Unfortunately, *models* are often incorrectly labeled as theories. For example, the 'theory' of the solar system developed since Copernicus and Kepler is a *model*, i.e. the application of a theory to a specific physical system, in this case, Newtonian gravitation *theory*. Another model of that same theory is the 'theory' of the tides.

The following may serve as an example of the occurrence of incommensurable terms. Let S be thermodynamics, and let T be statistical mechanics. In what sense does T imply S? While the mathematical laws of S are relatively easy to obtain from those of T, the *meaning* of the symbols involved, the semantics, poses a problem. 'Temperature' is a primary concept in thermodynamics and is operationally defined. It is also an intuitive notion through our sense perceptions of hot and cold. But in statistical mechanics which is based on the dynamics and interaction of molecules, such a notion does not exist on the fundamental level; in that theory, it is a derived notion. In the deduction of the fundamental laws of thermodynamics from those of statistical mechanics, a formal relationship between temperature and quantities primary to statistical mechanics can be found. That relationship can then be used to *define* temperature for statistical mechanics. In the very special case of the kinetic theory of gases to which the thermodynamics of uniform ideal gases is reduced, that relationship states that temperature is proportional to the average kinetic energy of the molecules. Clearly, temperature and kinetic energy are incommensurable terms.

The defining relations of concepts of S in terms of concepts in T (or *vice versa*) are called 'bridge laws'. Such relations are *forced* on us if theory reduction is to carry through. They are made *ad hoc*. There is no *logical* connection.

For the reasons just stated, it is clear that reduction of S to T does not involve a (logical) *deduction* of S from T, but simply establishes a connection between the two theories made possible by suitable bridge laws. It is therefore questionable whether this type of reduction has much significance: it does not permit one to say that S is *contained* in T and that therefore S is superfluous when T is given. We find it essential that different levels of theory have their autonomy and contribute in essential ways to our understanding of reality. They have qualities that distinguishes them from other levels.

The validity limits of a theory characterize its domain of applicability; they are given by inequalities of the form $p << 1$, such that p characterizes the error made by not applying the lower level theory: p is the dimensionless ratio of two physical quantities, $p = \frac{Q_1}{Q_2}$; by applying S to a given phenomenon one assumes Q_1 to be negligible compared to Q_2. For example, the validity limits of Newtonian mechanics involve the ratio of the (square of the) largest velocity, v, occurring in the phenomenon, to the (square of the) velocity of light, c, $p = \frac{v^2}{c^2}$. When S is reducible to T then S has a smaller domain than T.

A model M of a theory T has necessarily a domain less than that of T. But in addition, it involves various idealizations that further restrict its validity. It may also involve boundary and/or initial conditions specific to the case.

One can distinguish between two types of theory reductions: merological and non-merological ones. Merological reductions reduce the objects described by theory S to component objects described by T. Since the reduction of the theory of condensed matter to the theory of atoms is a prime example, this type of reduction has sometimes been dubbed: 'atomic reduction'. Other examples are the reduction of atomic nuclei (theory of nuclear physics) to quarks (theory of strong interactions), and the reduction of micro- organisms (microbiology) to molecules (molecular biology.)

But there are also reductions that do not involve the breakup of wholes into parts. The reduction of Newtonian gravitation theory to Einsteinian gravitation theory is a classical example of such a non-merological case (Rohrlich 1989). The reduction of Newtonian mechanics to special relativistic mechanics is another.

3 Reductive explanation

Reductive explanation is an *explanation* of a phenomenon or law on one level, L_S, by means of a theory (or part of a theory) from a lower (deeper) level L_T. No attempt is made to reduce the whole theory S to the whole theory T. The emphasis is here on *understanding*, the pragmatic goal of explanation. Reductive explanation is therefore a pragmatic activity. The bridge laws which in theory reduction are a source of difficulty because they break the logic of the argument are here welcome tools; they are used as postulates that help relate the deeper to the shallower level of understanding. An example will help to clarify this situation: superconductivity of a solid is a macroscopic phenomenon; its explanation requires the use of quantum mechanics on the lower (microscopic) level of the atomic and molecular structure of the solid. In this way the (macroscopic) 'phenomenon of superconductivity' is reduced to the (microscopic) 'theory of superconductivity'. The latter involves only a fraction of the theory of condensed matter, T.

It is unfortunate that the two concepts of theory reduction (as discussed above in (2)) and reductive explanation have been conflated for many years. This has brought a great deal of confusion to the problem of reduction, especially when 'reductive explanation' is misleadingly called 'explanatory reduction'. It is not a reduction at all; it does not reduce one theory to another theory, but it explains a phenomenon on one level by means of a theory (or part of a theory) on a lower level. Thus it accounts for macroscopic properties which are radically different from properties on a deeper level. It may therefore be more appropriately called a 'synthesis' in the merological case (atomistic reduction). Thus, Shimony (1993, vol. II, p.216) calls it 'the methodology of synthesis'.

In the non-merological case, 'synthesis' would not be appropriate; perhaps 'deeper understanding' may be a good characterization. Thus, for example, the phenomenon of black holes cannot be understood on the basis of Newtonian gravitation theory (NGT). The deeper understanding that Einsteinian gravitation theory (EGT) offers permits not only a (reductive) explanation of Kepler's laws and many other laws that *can* be explained by means of NGT, but also an explanation of black holes. These, therefore, *require* a reductive explanation. At the same time, it shows by example that EGT has a larger domain of validity than NGT.

Weinberg (1995) makes a distinction between 'grand reduction' and 'petty reduction'. To the present writers that distinction is approximately that of 'reductive explanation' and 'theory reduction'.

Reductive explanation is an essential part of scientific research; it involves none of the difficulties which beset theory reduction. As mentioned earlier, while bridge laws are still needed, these enter as *given relations* (postulates) that connect the given theory T to the phenomenon on level L_S which is to be explained. Exactly these relations are needed to make T understandable to someone who comes from L_S.

To be sure, it must be emphasized that *not all* explanations of phenomena on level L_S are of a reductive nature. Many phenomena on L_S can be satisfactorily explained by theory S without reduction, i.e. without reference to theory T. But others cannot and require recourse to a deeper level.

4 Reduction in Science

Having specified three different meanings of what can be meant by 'reduction': logical reduction, theory reduction and reductive explanation, we want to make some comments about the status of these different kinds of reduction in science.

As we have already mentioned, 'logical reduction' can only be used in mathematics or the mathematical components of a scientific theory. One of the main programs of the *logical positivists* culminating in the explicit research program of Carnap (1928), was the pursuit of the possibility of reducing all of science by logical reduction. Apart from the internal logical problems already hinted at, it is now accepted by philosophers of science that such a program is impossible. This is primarily because logical positivism is founded on too narrow a concept of human experience. It is, of course, by our experience that we acquire knowledge about the world; and this was correctly emphasized by the positivists. But the rational aspects of these experiences - those aspects that make us 'understand things' - cannot be entirely contained in a purely logical structure. This is were 'theory reduction' differs essentially from 'logical reduction'. Since the former necessarily relates theories from different levels, it uses *non-logical* propositions. These are the so called 'bridge-laws' mentioned earlier which relate different levels and which therefore - in a sense *ad hoc* - relate logically incompatible theories and models. As pointed out, these bridge-laws are essential in theory reduction and therefore preclude logical reduction as theory reduction. In fact, those who are used to the word 'reduction' as used in logical reduction may not consider 'theory reduction' to be a reduction at all. Finally, 'reductive explanation' goes one step further and uses bridge-laws not to connect two theories from two different levels, but as a given tool to *explain* a particular phenomenon or law *(i.e. not a theory)* that occurs on a higher level by means of a theory that is granted to exist on a lower (deeper) level. It is this process of reductive explanation that Weinberg *(loc.cit.)* calls 'grand reductionism'. And he himself emphasizes the 'explanatory' over the 'reductive' element. He says "From Newton's time to our own we have seen a steady expansion of the range of phenomena that we know how to *explain*, and a steady improvement in the simplicity and universality of the theories used in these *explanations*. Science in this style is properly called reductionist."*(loc. cit.*p.39, italics ours). This is exactly what we have called 'reductive explanation', and what Weinberg calls 'grand reductionism'.

Indeed, the search for understanding and explanation is the driving force of all science, and it should be cherished and treated with respect. Reductive explanation is the main activity of scientists; it is what they do in their everyday work. They do not do 'theory reduction'. 'Theory reduction' is of much more interest to philosophers of science. That the former, reductive explanation, is more interesting to scientists should not be surprising because 'explanation' is a *pragmatic* activity and scientists are pragmatists.

When a whole new theory is developed (by far not an every event), the mathematical component of that theory must be such that the 'old theory' is mathematically reducible to it. This is demanded by the fact that no sharp boundary can be drawn between two different levels. Insurance of this reducibility is the only activity of the theoretical scientist which resembles theory reduction.

But theory reduction involves a lot more than that: care must be taken of the meaning attributed to the symbols in the theories. Theories involve symbols whose *meaning changes* in the reduction process. This meaning change is 'obvious' to the scientists and no further attention is paid to it by the scientist who is only interested in the mathematical component of theory reduction (Rohrlich 1989). The philosopher of science, however, pays considerable attention to meaning change. This is one source for the conflation by scientists of theory reduction and reductive explanation.

The most important point of both 'theory reduction' and 'reductive explanation' is that *they relate to different ontologies* (Aerts 1995, 1998). One often refers to the old (higher level) ontology as *wrong* and to the new (lower level) one as *correct*. In fact, sometimes the older *theory* is said to be false and the new one correct. For example, Newtonian mechanics is said to be 'false', and special relativistic mechanics is said to be 'correct'. But this is a narrow view: is not, by the same token, special relativistic mechanics also 'false' and general relativistic mechanics 'correct'? And will not the latter be soon replaced by the mechanics of quantum gravity? A consistent application of calling the higher level description 'wrong' and the lower one 'correct' therefore leads to the conclusion that all theories are wrong except the 'final theory' - if there exists one.

A more appropriate approach would be to call every mature theory 'correct' meaning 'correct within the applicability domain of the theory'. The latter is a well defined concept once the next lower level theory is known. In scientific praxis this is indeed always done. Just imagine calculating the trajectory of a thrown ball by means of general relativity theory rather than by Newtonian mechanics (which - after all - is false)! Thus one can argue that the theories on each level have a certain autonomy (within limits) despite their reducibility (Rohrlich 1995). But this issue leads into the deeper question of the nature of reality and is outside the present topic of discussion. However, the following remarks must be made here.

There is a good reason to believe that we know more about the nature of reality, that we have better explanations, and that we have a deeper understanding as a result of reduction. But reduction is very intimately related to the view of a layered structure of reality with each layer giving a *qualitative different* description of it! (Rohrlich 1988, Aerts 1995, 1998). Which description is correct? This is to be determined *pragmatically*. It depends on many things: on the scale with which we want to look at a particular piece of the world, on the question asked, and on the sofistication of the questioner who is seeking an explanation.

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