The creation-discovery-view : towards a possible explanation of quantum reality*

Diederik Aerts and Bob Coecke

CLEA, Free University of Brussels, Pleinlaan 2, 1050 Brussels, Belgium.
e-mails: diraerts@vub.ac.be, bocoecke@vub.ac.be

Abstract

We present a realistic interpretation for quantum mechanics that we have called the 'creation discovery view' and that is being developed in our group in Brussels. In this view the change of state of a quantum entity during an experiment is taken to be a 'real change' under influence of the experiment, and the quantum probability that corresponds to the experiment is explained as due to a lack of knowledge of a deeper deterministic reality of the measurement process. The technical mathematical theory underlying the creation discovery view that we are elaborating we have called the 'hidden measurement formalism'. We present a simple physical example: the 'quantum machine', where we can illustrate easily how the quantum structure arises as a consequence of the two mentioned effects, a real change of the state, and a lack of knowledge about a deeper reality of the measurement process. We analyze non-locality in the light of the creation discovery view, and show that we can understand it if we accept that also the basic concept of 'space' is partly due to a creation: when a detection of a quantum entity in a non-local state occurs, the physical act of detection itself 'creates' partly the 'place' of the quantum entity. In this way the creation discovery view introduces a new ontology for space: space is not the all embracing theater, where all 'real' objects have their place, but it is the structure that governs a special type of relations (the space-like relations) between macroscopic physical entities. We bring forward a number of elements that show the plausibility of the approach and also analyze the way in which the presence of Bell-type correlated events can be incorporated.

*Published as: Aerts, D. and Coecke, B., 1999, "The creation-discovery-view : towards a possible explanation of quantum reality", in *Language, Quantum, Music*, eds. Dalla Chiara, M.L., Kluwer Academic, Dordrecht.

1 Introduction

The creation discovery view and together with it its technically underlying hidden measurement formalism has been elaborated from the early eighties on, and many aspects of it have been exposed in different places [6, 7, 12, 13, 15, 16, 19, 20, 22, 23, 30, 31, 32, 33, 34, 35, 36]. In this paper we give an overview of the most important of these aspects.

Quantum mechanics was originally introduced as a non-commutative matrix calculus of observables by Werner Heisenberg [41] and parallel as a wave mechanics by Erwin Schrödinger [43]. These two structurally very different theories could explain fruitfully the early observed quantum phenomena. Already in the same year the two theories were shown to be realizations of the same, more abstract, ket-bra formalism by Dirac [38]. Only some years later, in 1934, John Von Neumann put forward a rigorous mathematical framework for quantum theory in an infinite dimensional separable complex Hilbert space [46]. Matrix mechanics and wave mechanics appear as concrete realizations: the first one if the Hilbert space is l^2, the collection of all square summable complex numbers, and the second one if the Hilbert space is L^2, the collection of all square integrable complex functions. The formulation of quantum mechanics in the abstract framework of a complex Hilbert space is now usually referred to as the 'standard quantum mechanics'.

The basic concepts - the vectors of the Hilbert space representing the states of the system and the self-adjoint operators representing the observables - in this standard quantum mechanics are abstract mathematical concepts defined mathematically in and abstract mathematical space. Several approaches have generalized the standard theory starting from more physically defined basic concepts. John Von Neumann and Garett Birkhoff have initiated one of these approaches [29] were they analyze the difference between quantum and classical theories by studying the 'experimental propositions'. They could show that for a given physical system classical theories have a Boolean lattice of experimental propositions while for quantum theory the lattice of experimental propositions is not Boolean. Similar fundamental structural differences between the two theories have been investigated by concentrating on different basic concepts. The collection of observables of a classical theory was shown to be a commutative algebra while this is not the case for the collection of quantum observables [40, 44]. Luigi Accardi and Itamar Pitowski obtained an analogous result by concentrating on the probability models connected to the two theories: classical theories have a Kolmogorovian probability model while the probability model of a quantum theory is non Kolmogorovian [1, 42].

The fundamental structural differences between the two types of theories, quantum and classical, in different categories, was interpreted as indicating also a fundamental difference on the level of the nature of the reality that both theories describe: the micro world should be 'very different' from the macro world. This state of affairs was very convincing also because concrete attempts to understand quantum mechanics in a classical way had failed as well: e.g. the many 'physical' hidden variable theories that had been tried out [45]. The structural difference between quantum theories and classical theories (Boolean lattice versus non- Boolean lattice of propositions, commutative algebra versus non commutative algebra of observables and Kolmogorovian versus non Kolmogorovian probability structure) had been investigated mostly mathematically and not much understanding of the physical meaning of the structural differences had been gained during all these years.

The first step that led to the creation discovery view and its underlying hidden measurement formalism was a breakthrough in the understanding of the physical origin of these mathematical structural differences between quantum and classical theories. Indeed, one of the authors found in the early eighties a way to identify the physical aspects that are at the origin of the structural differences [3, 6, 7]. Let us summarize these findings: it are mainly two aspects that determine the mathematical structural differences between classical and quantum theories in the different categories:

We have a quantum-like theory describing a system under investigation if the measurements needed to test the properties of the system are such that:

(1) The measurements are not just observations but provoke a real change of the state of the system.

(2) There exists a lack of knowledge about the reality of what happens during the measurement process.

The presence of these two aspects is sufficient to render the description of the system under consideration quantum-like. It is the lack of knowledge (2) that is theoretically structured in a non Kolmogorovian probability model. In a certain sense it is possible to interpret the second aspect, the presence of the lack of knowledge on the reality of the measurement process, as the presence of 'hidden measurements' instead of 'hidden variables'. Indeed, if a measurement is performed with the presence of such a lack of knowledge, then this is actually the classical mixture of a set of classical hidden measurements, were for such a classical hidden measurement there would be no lack of knowledge. In an analogous way as in a hidden variable theory, the quantum state is a classical mixture of classical states. This is the reason why we have called the underlying theory of the creation discovery view the hidden measurement formalism. It is possible to illustrate the creation discovery view and the hidden measurement aspect in a very simple way by using a mechanical model that was introduced in [5, 16, 7] and that we have called the quantum machine. This is the subject of next section.

2 The Quantum Machine.

Several aspects of the quantum machine have been presented in different occasions [6, 7, 8, 9, 10, 15, 19, 20] and we shall therefore introduce here only the basic aspects. The machine that we consider consists of a physical entity S that is a point particle P that can move on the surface of a sphere, denoted $surf$, with center O and radius 1. The unit-vector v where the particle is located on $surf$ represents the state p_v of the particle (see Fig 1,a). For each point $u \in surf$, we introduce the following measurement e_u. We consider the diametrically opposite point $-u$, and install a piece of elastic of length 2, such that it is fixed with one of its end-points in u and the other end-point in $-u$. Once the elastic is installed, the particle P falls from its original place v orthogonally onto the elastic, and sticks on it (Fig 1,b). Then the elastic breaks and the particle P, attached to one of the two pieces of the elastic (Fig 1,c), moves to one of the two end-points u or $-u$ (Fig 1,d). Depending on whether the particle P arrives in u (as in Fig 1) or in $-u$, we give the outcome o_1^u or o_2^u to e_u. We can easily calculate the probabilities corresponding to the two possible outcomes.

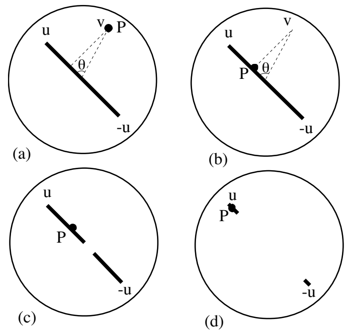

Fig 1 : A representation of the quantum machine. In (a) the physical entity P is in state p_v in the point v, and the elastic corresponding to the measurement e_u is installed between the two diametrically opposed points u and $-u$. In (b) the particle P falls orthogonally onto the elastic and stick to it. In (c) the elastic breaks and the particle P is pulled towards the point u, such that (d) it arrives at the point u, and the measurement e_u gets the outcome o_1^u.

The particle P arrives in u when the elastic breaks in a point of the interval L_1 (which is the length of the piece of the elastic between $-u$ and the point where the particle has arrived, or $1 + cos\theta$), and arrives in $-u$ when it breaks in a point of the interval L_2 ($L_2 = L - L_1 = 2 - L_1$). We make the hypothesis that the elastic breaks uniformly, which means that the probability that the particle, being in state p_v, arrives in u, is given by the length of L_1 divided by the length of the total elastic (which is 2). The probability that the particle in state p_v arrives in $-u$ is the length of L_2 (which is $1 - cos\theta$) divided by the length of the total elastic. If we denote these probabilities respectively by $P(o_1^u, p_v)$ and $P(o_2^u, p_v)$ we have:

$$P(o_1^u, p_v) = \frac{1 + cos\theta}{2} = cos^2\frac{\theta}{2} \qquad P(o_2^u, p_v) = \frac{1 - cos\theta}{2} = sin^2\frac{\theta}{2} \qquad (1)$$

These transition probabilities are the same as the ones related to the outcomes of a Stern-Gerlach spin measurement on a spin $\frac{1}{2}$ quantum particle, of which the quantum-spin-state in direction $v = (cos\phi sin\theta, sin\phi sin\theta, cos\theta)$, denoted by $\bar{\psi}_v$, and the measurement e_u corresponding to the spin measurement in direction $u = (cos\beta sin\alpha,\ sin\beta sin\alpha, cos\alpha)$, is described respectively by the vector and the self adjoint operator of a two-dimensional complex Hilbert space.

$$\psi_v = (e^{-i\phi/2}cos\theta/2, e^{i\phi/2}sin\theta/2) \qquad\qquad H_u = \frac{1}{2}\begin{pmatrix} cos\alpha & e^{-i\beta}sin\alpha \\ e^{i\beta}sin\alpha & cos\alpha \end{pmatrix} \quad (2)$$

We can easily see now the two aspects in this quantum machine that we have identified in the creation discovery view to give rise to the quantum structure. The state of the particle P is effectively changed by the measuring apparatus (p_v changes to p_u or to p_{-u} under the influence of the measuring process), which identifies the first aspect, and there is a lack of knowledge on the interaction between the measuring apparatus and the particle, namely the lack of knowledge of were exactly the elastic will break, which identifies the second aspect. We can also easily understand now what is meant by the term 'hidden measurements'. Each time the elastic breaks in one specific point λ, we could identify the measurement process that is carried out afterwards as a hidden measurement e_u^λ. The measurement e_u is then a classical mixture of the collection of all measurement e_u^λ: namely e_u consists of choosing at random one of the e_u^λ and performing this chosen e_u^λ. We can see in the example that effectively the final states, after the measurement e_u, hence the states p_u and p_{-u}, are partly created by the measurement.

We remark that we have shown in our group in Brussels that such a model can be built for any arbitrary quantum entity [5, 6, 7, 14, 30, 31, 32, 35, 36]. However, the hidden measurement formalism is more general than standard quantum theory. Indeed, it is very easy to produce quantum-like structures that cannot be represented in a complex Hilbert space. For a presentation of an as general as possible mathematical framework to deal with the hidden measurement ideas we refer to [32, 35, 36]. An example of such quantum-like structures is presented in the following section where we define a continuous transition from quantum to classical for the model introduced at the beginning of this section. Hence we claim to solve the old problem of the classical limit.

3 The Quantum Classical Transition.

If the quantum structure can be explained by the presence of a lack of knowledge on the measurement process, we can go a step further, and wonder what types of structure arise when we consider the original models, with a lack of knowledge on the measurement process, and introduce a variation of the magnitude of this lack of knowledge. We have studied the quantum machine under varying 'lack of knowledge', parameterizing this variation by a number $\epsilon \in [0, 1]$, such that $\epsilon = 1$ corresponds to the situation of maximal lack of knowledge, giving rise to a quantum structure, and $\epsilon = 0$ corresponds to the situation of zero lack of knowledge, generating a classical structure. Other values of ϵ correspond to intermediate situations and give rise to a structure that is neither quantum nor classical [22, 23, 24, 25]. We have called this model the ϵ-model and introduce it shortly.

a) The ϵ-Model. We start from the quantum machine, but introduce now different types of elastic. An ϵ, d-elastic consists of three different parts: one lower part where it is unbreakable, a middle part where it breaks uniformly, and an upper part where it is again unbreakable. By means of the two parameters $\epsilon \in [0, 1]$ and $d \in [-1+\epsilon, 1-\epsilon]$, we fix the sizes of the three parts in the following way. Suppose that we have installed the ϵ, d-elastic between the points $-u$ and u of the sphere. Then the elastic is unbreakable in the lower part from $-u$ to $(d-\epsilon) \cdot u$, it breaks uniformly in the part from $(d-\epsilon) \cdot u$ to $(d+\epsilon) \cdot u$, and it is again unbreakable in the upper part from $(d+\epsilon) \cdot u$ to u (see Fig 2).

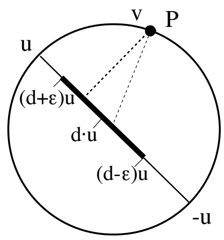

Fig 2 : A representation of the measurement $e^{\epsilon}_{u,d}$. The elastic breaks uniformly between the points $(d-\epsilon)u$ and $(d+\epsilon)u$, and is unbreakable in other points.

An e_u measurement performed by means of an ϵ, d- elastic shall be denoted by $e^{\epsilon}_{u,d}$. We have the following cases:

(1) $v \cdot u \leq d - \epsilon$. The particle sticks to the lower part of the ϵ, d-elastic, and any breaking of the elastic pulls it down to the point $-u$. We have $P^{\epsilon}(o^u_1, p_v) = 0$ and $P^{\epsilon}(o^u_2, p_v) = 1$.

(2) $d - \epsilon < v \cdot u < d + \epsilon$. The particle falls onto the breakable part of the ϵ, d-elastic. We can easily calculate the transition probabilities and find:

$$P^{\epsilon}(o^u_1, p_v) = \frac{1}{2\epsilon}(v \cdot u - d + \epsilon) \qquad P^{\epsilon}(o^u_2, p_v) = \frac{1}{2\epsilon}(d + \epsilon - v \cdot u) \qquad (3)$$

(3) $d + \epsilon \leq v \cdot u$. The particle falls onto the upper part of the ϵ, d-elastic, and any breaking of the elastic pulls it upwards, such that it arrives in u. We have $P^{\epsilon}(o^u_1, p_v) = 1$ and $P^{\epsilon}(o^u_2, p_v) = 0$.

We are now in a very interesting situation from the point of view of the structural studies of quantum mechanics. Since the ϵ-model describes a continuous transition from quantum to classical, its mathematical structure should be able to learn us *what are the structural shortcomings of the standard Hilbert space quantum mechanics.* Therefore we have studied the ϵ-model in the existing mathematical approaches that are more general than the standard quantum mechanics: the lattice approach, the probabilistic approach and the *-algebra approach.

b) The Lattice Approach. In this lattice approach exists a well known axiomatic scheme that reduces the approach to standard quantum mechanics if certain axioms are fulfilled. For intermediate values of ϵ, that is $0 < \epsilon < 1$, we find that 2 of the 5 axioms needed in the lattice approach to reconstruct standard quantum mechanics are violated. The axioms that are violated are the weak-modularity and the covering law, and it are precisely those axioms that are needed to recover the vector space structure of the state space in quantum mechanics [22, 23, 25].

c) The Probabilistic Approach. If we take the case of vanishing fluctuations ($\epsilon = 0$), do we obtain the Kolmogorovian theory of probability? This would be most interesting, since then we would have constructed a macroscopic model with an understandable structure (i.e., we can see how the probabilities arise) and a quantum and a classical behavior. We [15] proposed to test the polytopes for a family of conditional probabilities. The calculations can be found in [26] and the result was what we hoped for: a macroscopic model with a quantum and a Kolmogorovian limit. For intermediate values of the fluctuations ($0 < \epsilon < 1$) the resulting probability model is neither quantum nor Kolmogorovian: we have identified here a new type of probability model, that is quantum-like, but not really isomorphic to the probability model found in a complex Hilbert space.

*d) The *-Algebra Approach.* The *-algebra provides a natural mathematical language for quantum mechanical operators. We applied the concepts of this approach to the epsilon model to find that an operator corresponding to an ϵ measurement is linear if and only if $\epsilon = 1$ [21]. This means that for the classical and intermediate situations the observables cannot be described by linear operators.

CONCLUSION: Quantum theory and classical theories appear as special cases ($\epsilon = 1, \epsilon = 0$*) and the general intermediate case, although quantum- like, cannot be described in standard Hilbert space quantum mechanics.*

e) The Measurement Problem and the Schrödingers Cat Paradox. The result stated in the conclusion means that all the paradoxes of standard quantum theory that are due to the fact that quantum theory is used as a universal theory, also being applied to macroscopic system, for example the measuring apparatus, are not present in our hidden measurement formalism. We explicitly have in mind the 'measurement problem' and the 'Schrödinger cat paradox'. Indeed, the measurement apparatus should be described by a classical model in our approach, and the physical system eventually by a quantum model. The problem of the presence of quantum correlations between physical system and measuring apparatus, as it presents itself in the standard theory, takes a completely different aspect. We are working now at the elaboration of a concrete description of the measurement process within the hidden measurement formalism [23]. Our result also shows that it is possible in the hidden measurement formalism to formulate a 'classical limit', namely as a continuous transition from quantum to classical [25].

4 Non-Locality: a Genuine Property of Nature.

The measurement problem and paradoxes equivalent to Schrödingers cat paradox disappear in the hidden measurement formalism, because standard quantum mechanics appears only as a special case, the situation of maximum fluctuations. All quantum mysteries connected to the effect of 'non-locality' remain (Einstein Podolsky Rosen paradox and the violation of the Bell inequalities). It is even so that non-locality unfolds itself as a fundamental aspect of the creation discovery view. This is due to the fact that if we explain the quantum structures as it is done in this view, a quantum measurement has two concrete physical effects: (1) it changes the state of the system and (2) it produces probability due to a lack of knowledge about the nature of this change. With the quantum machine we have given a macroscopic model for a spin measurement of a spin $\frac{1}{2}$ particle. If we apply the hidden measurement formalism to the situation of a quantum system described by a wave function $\psi(x)$, and to a position or a momentum measurement performed on this system, we also have the two mentioned effects. For example in the case of a position measurement: the detection apparatus changes the state of the quantum system, in the sense that it 'localizes' the quantum system in a specific place of space, and the probabilities that are connected with this measurement are due to the fact that we have a lack knowledge about the specific way in which this localization takes place. This means that 'before the detection has taken place' the quantum system was in general not localized: it was not present in a specific region of space. With other words, quantum systems are fundamentally non-local systems: the wave function $\psi(x)$ describing such a quantum state is not interpreted as a wave that is present in space, but as indicating these regions of space were the particle can be localized, where $\int_R |\psi(x)|^2 dx$ is the probability that this localization will happen in region R. A similar interpretation must be given to the momentum measurement of a quantum entity: the quantum entity has no momentum before the measurement, but the measurement creates partly this momentum. In [25] we have calculated the ϵ- situation for a quantum system described by a wave function $\psi(x)$, element of the Hilbert space of all square integrable complex functions, and we have found the following very simple procedure. Suppose that ϵ is given, and the state of the quantum system is described by the wave function $\psi(x)$ and $\phi(x)$ is the corresponding probability distribution (hence $\phi(x) = |\psi(x)|^2$). We cut, by means of a constant function ϕ_Ω, a

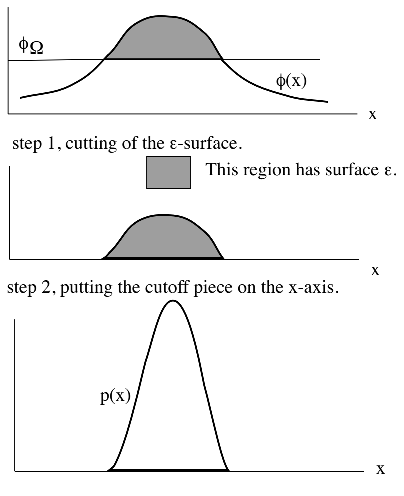

Fig 3 : step 3, renormalizing by dividing by ε.

piece of the function $\phi(x)$, such that the surface contained in the cutoff piece equals ϵ (see step 1 of Fig 3). We move this piece of function to the x-axis (see step 2 of Fig 3). And then we renormalize by dividing by ϵ (see step 3 of Fig 3). If we proceed in this way for smaller values of ϵ, we shall finally arrive at a delta-function for the classical limit $\epsilon \rightarrow 0$, and the delta-function is located in the original maximum of the quantum probability distribution. We want to point out that the state $\psi(x)$ of the physical system is not changed by this ϵ-procedure, it remains always the same state, representing the same physical reality. It is the regime of lack of knowledge going together with the detection measurement that changes with varying ϵ. For $\epsilon = 1$ this regime is one of maximum lack of knowledge on the process of localization, and this lack of knowledge is characterized by the spread of the probability distribution $\phi(x)$. For an intermediate value of ϵ, between 1 and 0, the spread of the probability distribution has decreased (see Fig 3) and for zero fluctuations the spread is 0. Let us also try to see what becomes of the non-local behavior of quantum entities taking into account the classical limit procedure that we propose. Suppose that we consider a double slit experiment, then the state p of a quantum entity having passed the slits can be represented by a probability function $p(x)$ of the form represented in Fig 4. We can see that the non-locality presented by this probability function gradually disappears when ϵ becomes smaller, and in the case where $p(x)$ has only one maximum finally disappears completely. When there are no fluctuations on the measuring apparatus used to detect the particle, it shall be detected with certainty in one of the slits, and always in the same one. If $p(x)$ has two maxima (one behind slit 1, and the other behind slit 2) that are equal, the non-locality does not disappear. Indeed, in this case the limit- function is the sum of two delta-functions (one behind slit 1 and one behind slit 2). So in this case the non-locality remains present even in the classical limit. If our procedure for the classical limit is a correct one, also macroscopic classical entities can be in non-local states. How does it come that we don't find any sign of this non-locality in the classical macroscopic world? This is due to the fact that the set of states, representing a situation where the probability function has more than one maximum, has measure zero, compared to the set of all possible states, and moreover these states are 'unstable'. The slightest perturbation will destroy the symmetry of the different maxima, and hence shall give rise to one point of localization in the classical limit. Also classical macroscopic reality is non-local, but the local model that we use to describe it gives the same statistical results, and hence cannot be distinguished from the non-local model.

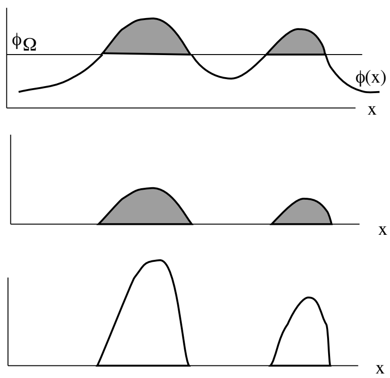

Fig 4 : the classical limit procedure in the situation of a non-local quantum state.

5 Bell Inequalities and Hidden Correlations.

It is interesting to consider the violation of Bell's inequalities within the creation discovery view. The quantum machine, as presented in section 3, delivers us a macroscopic model for the spin of a spin $\frac{1}{2}$ quantum entity and starting with this model it is possible to construct a macroscopic situation, using two of these models coupled by a rigid rod, that represents faithfully the situations of two entangled quantum particles of spin $\frac{1}{2}$ [11]. The 'non-local' element is introduced explicitly by means of a rod that connects the two sphere-models. We also have studied this EPR situation of entangled quantum systems by introducing the ϵ-variation of the amount of lack of knowledge on the measurement processes and could show that one violates the Bell-inequalities even more for classical but non-locally connected systems, that is, $\epsilon = 0$. This illustrates that the violation of the Bell- inequalities is due to the non-locality rather then to the indeterministic character of quantum theory. And that the quantum indeterminism (for values of ϵ greater than 0) tempers the violation of the Bell inequalities [18]. This idea has been used by one of the authors to construct a general representation of entangled states (hidden correlations) within the hidden measurement formalism [32, 33, 34]. More precisely, it is possible to prove that for a collection of quantum entities described in a tensor product of Hilbert spaces there always exists one unique hidden correlation representation, i.e., a representation as a collection of individual entities which are correlated in such a way that a change of state of one of the entities (due to a measurement) induces in a well-defined change of state of the other entities (for more details we refer to [34]). Structurally, these correlations are definitely mechanistic. Nonetheless, they are non-local.

References

[1] Accardi, L. (1982). *Nuovo Cimento*, **34**, 161.

[2] Accardi, L. (1984). *The Probabilistic Roots of the Quantum Mechanical Paradoxes*, in *The Wave-Particle Dualism*, eds Diner, S. et al., Reidel Publishing Company, Dordrecht.

[3] Aerts, D. (1982). *Lett. Nuovo Cimento*, **34**, 107.

[4] Aerts, D. (1984). *How do we have to change Quantum Mechanics in order to describe Separated Systems*, in *The Wave-Particle Dualism*, eds Diner, S. et al., Reidel Publishing Company, Dordrecht.

[5] Aerts, D. (1985). *A Possible Explanation for the Probabilities of Quantum Mechanics and a Macroscopic Situation that Violates Bell Inequalities*, in *Recent Developments in Quantum Logic*, eds. Mittelstaedt, P. et al., in *Grundlagen der Exacten Naturwissenschaften*, vol.6, Wissenschaftverlag, Bibliographisches Institut, Mannheim, 235.

[6] Aerts, D. (1986). *J. Math. Phys.* **27**, 202.

[7] Aerts, D. (1987). *The Origin of the Non-Classical Character of the Quantum Probability Model*, in *Information, Complexity, and Control in Quantum Physics*, eds. Blanquiere, A. et al., Springer-Verlag.

[8] Aerts, D., (1988a). *The Physical Origin of the EPR Paradox and How to Violate Bell Inequalities by Macroscopic Systems*, in *Symposium on the Foundations of Modern Physics*, eds. Lahti, P. et al., World Scientific, Singapore.

[9] Aerts, D., (1988b). *The Description of Separated Systems and a Possible Explanation of the Probabilities of Quantum Mechanics*, in *Microphysical Reality and Quantum Formalism*, eds. van der Merwe et al., Kluwer Academic Publishers.

[10] Aerts, D. (1991a). *A Macroscopic Classical Laboratory Situation with Only Macroscopic Classical Entities giving rise to a Quantum Mechanical Probability Model*, in *Quantum Probability and Related Topics, Vol. VI,* ed. Accardi, L., World Scientific, Singapore.

[11] Aerts, D. (1991b).*Helv. Phys. Acta*, **64**, 1.

[12] Aerts, D. (1992). *Int. J. Theor. Phys.*, **31**, 1815.

[13] Aerts, D. (1993). *De Muze van het Leven: Quantummechanica en de Aard van de Werkelijkheid*, Pelckmans, Kapellen, Belgium (in this book the creation discovery view is exposed in its most complete form, an English translation is being prepared).

[14] Aerts, D. (1994). *Found. Phys.*, **24**, 1227.

[15] Aerts, D. (1995). *Int. J. Theor. Phys.*, **34**, 1165.

[16] Aerts, D. (1996). *The hidden-measurement-formalism: what can be explained and where quantum paradoxes remain*, *Int. J. Theor. Phys.*, **36**, to appear.

[17] Aerts, D. and Aerts, S. (1996) *The Hidden Measurement approach to Quantum Mechanics*, in *New Developments on Fundamental Problems in Quantum Physics*, eds. Ferrero, M. and Van der Merwe, A., Kluwer Academic, Dordrecht.

[18] Aerts, D., Aerts, S., Coecke, B. and Valckenborgh, F. (1996). *The Meaning of the Violation of Bell's Inequalities: Non-Local Correlation or Quantum Behavior?*, Preprint CLEA, Free University of Brussels.

[19] Aerts, D., Coecke, B., Durt, T. and Valckenborgh, F. (1996a). *Quantum, Classical and Intermediate; a Model on the Poincaré Sphere*, in the proceedings of the Winterschool on Measure Theory, Liptovski, 1995, to be published in *Tatra Mountains Mathematical Publications.*

[20] Aerts, D., Coecke, B., Durt, T. and Valckenborgh, F. (1996b). *Quantum, Classical and Intermediate; the Vanishing Vector Space Structure*, in the proceedings of the Winterschool on Measure Theory, Liptovski, 1995, to be published in *Tatra Mountains Mathematical Publications.*

[21] Aerts, D. and D'Hooghe, B. (1996). *Operator structure of a non- quantum and a non-classical system*, *Int. J. Theor. Phys.*, **35**, to appear.

[22] Aerts, D. and Durt, T. (1994a). *Found. Phys.* **24**, 1353.

[23] Aerts, D and Durt, T. (1994b), *Quantum, Classical and Intermediate; a measurement model*, in the proceedings of the *Symposium on the Foundations of Modern Physics,* Helsinki, 1994, ed. K.V. Laurikainen, C. Montonen, K. Sunnar Borg, Editions Frontieres, Gives sur Yvettes, France.

[24] Aerts, D., Durt, T. and Van Bogaert, B. (1992). *Tatra Montains Math. Publ.* **1**, 5.

[25] Aerts, D., Durt, T. and Van Bogaert, B. (1993). *Quantum Probability, the Classical Limit and Non-Locality*, in *Symposium on the Foundations of Modern Physics*, ed. Hyvonen, T., World Scientific, Singapore.

[26] Aerts, S. (1996) *Conditional Probabilities with a Quantal and a Kolmogorovian Limit*, *Int. J. Theor. Phys.*, **35**, to appear.

[27] Bell, J. (1964). *Physics*, **1**, 195.

[28] Bohm, D., (1951). *Quantum Theory*, Prentice-Hall.

[29] Birkhoff, G. and Von Neumann, J. (1936). *J., Ann. Math.*, **37**, 823.

[30] Coecke, B., (1995a). *Int. J. Theor. Phys.*, **34**, 1313.

[31] Coecke, B., (1995b). *Found. Phys. Lett.*, **8**, 447.

[32] Coecke, B., (1995c). *Hidden Measurement Systems*, Doctoral Dissertation, Free University of Brussels.

[33] Coecke, B., (1996a). *Int. J. Theor. Phys.*, **35**, 2371.

[34] Coecke, B., (1996b). *A Representation for Compound Quantum Systems as Individual Entities: Hidden Correlations*, Preprint CLEA, Free University of Brussels.

[35] Coecke, B., (1996b). *Classical Representations for Quantum-like Systems through an Axiomatics for Context Dependence*, *Helv. Phys. Acta*, **69**, to appear.

[36] Coecke, B., (1996b). *A Classification of Classical Representations for Quantum-like Systems*, *Helv. Phys. Acta*, **69**, to appear.

[37] Coecke, B., D'Hooghe, B. and Valckenborgh, F. (1996) *Classical Physical Entities with a Quantum Description*, in *New Developments on Fundamental Problems in Quantum Physics*, eds. Ferrero, M. and Van der Merwe, A., Kluwer Academic, Dordrecht.

[38] Dirac, P.A.M.(1958). *The Principles of Quantum Mechanics*, Clarendon Press, Oxford.

[39] Einstein, A., Podolsky, B. and Rosen, N. (1935). *Phys. Rev.*, **47**, 777.

[40] Emch, G. G. (1984), *Mathematical and Conceptual Foundations of 20th Century Physics,* North-Holland, Amsterdam.

[41] Heisenberg, W.(1925). *Zeitschr. Phys.* **33**, 879.

[42] Pitovski, I. (1989). *Quantum Probability - Quantum Logic*, Springer Verlag.

[43] Schrödinger, E.(1926). *Ann. der Phys.* **79**, 1926.

[44] Segal, I. E. (1947). *Ann. Math.*, **48**, 930.

[45] Selleri, F.(1990). *Quantum Paradoxes and Physical Reality*, Kluwer Academic Publishers, Dordrecht.

[46] Von Neumann, J.(1955). *Mathematical Foundations of Quantum Mechanics*, Princeton University Press, Princeton.