On the Origin of Probabilities in Quantum Mechanics: Creative and Contextual Aspects*

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1 Introduction

At the beginning of this century, a lot of changes as well in the field of politics, art and sciences have led to a change of paradigms and ways in which people think and interact with '(the/their) world'. That climate of change lasted the whole century, due to the inertia of old ideas and the required time people needed to build 'new images' (or even, new world views) that incorporate the new findings. Nonetheless, as soon as old paradigms get overruled, new ones appear. This is something which also happened after the 'invention' of quantum physics. The modernist deterministic world guided by Laplace's prime intelligence had to make place for one in which appear *probabilities that have a mysterious status*, as we will explain now. The probabilities of classical statistical theories [1, 3], e.g., statistical, mechanics, thermodynamics, classical probability calculus, have never been considered to be an obscure subject, because they can be explained as being due to a lack of knowledge about an eventual deterministic underlying reality. So, these classical probabilities are only a mathematical formalization of the lack

*Published as: Aerts, D., 1999, "From freedom to freedom of motion", in *A World in Transition, Humankind and Nature*, eds. Aerts, D., Broekaert, J. and Wijns, W., Kluwer Academic, Dordecht.

of knowledge about the system under study. When quantum mechanics was born as an intrinsic probabilistic theory, the question was raised rapidly of whether these quantum probabilities [5, 10, 13, 17] can also be explained as due to a lack of knowledge. The field of research investigating this problem was referred to as the search for hidden variable theories, the hidden variables describing this so called deterministic underlying reality. During the years many theorems (e.g., the famous no-go theorem of J. von Neumann [4], or its elaborations [6, 7, 8].) have shown that hidden variable theories for quantum mechanics are impossible, indicating that quantum probabilities are of a fundamentally different nature than classical probabilities and seemingly not due to a lack of knowledge. Some physicists formulated very clearly their opinion: quantum mechanical probabilities are ontologically present in reality itself. These ontological (or objective) probabilities destroyed the *classical picture* of the world in such a way that the search for an image of what really happens in the 'physical world' had been abandoned, and still is so in many fields of micro-physics. As such, a large quantity of the contemporary community of physicists consider 'real' physics as something definitely complementary to anything to be understood as possibly eligible by 'realism'. Without going into any debate on this, we will show that it is indeed possible to find a picture of quantum entities where these 'strange' probabilities are explained. In fact, even formally, we only have to introduce a specific new concept within the theory of physics, that was not explicitly present before, namely a model of *aspects of creation*.

More explicitly, when we consider a measurement on a physical entity, these aspects of creation can be modeled as an interaction of the physical entity with its measurement environment, in such a way that a measurement on this entity provokes a *change of state* of the entity that depends on the interaction with its measurement context. As it is explicitly shown in [15, 16], there are *two* aspects that determine structural differences, in the sense that we obtain quantum-like probability structures, if the measurements needed to test the properties of the system are such that:

1. *The measurements are not just observations but provoke a real change of the state of the system.*

2. *There exists a lack of knowledge on what precisely happens during the measurement process.*

The first aspect, the change of state, can be interpreted as an 'act of creation' on the entity under study. It is indeed the external device that provokes the change of state during the interaction with the entity. If there is not such a change of state, we can consider the measurement as a discovery. The second aspect, the presence of the lack of knowledge on the precise act of creation which results from an interaction with the measurement context, lies at the origin of the so called *indeterministic nature* of quantum measurements and can be formalized as a lack of knowledge on the precise measurement that is actually performed [15, 16]. We can formalize the foregoing ideas somewhat more concretely in the following way:

1. *With each real measurement* e *corresponds a collection of deterministic measurements* e_λ*, called 'hidden measurements'.*

2. *When a measurement* e *is performed on an entity in a state* p*, then one of the hidden measurements* e_λ *takes place. The probability finds its origin in the lack of knowledge about which one of the hidden measurements effectively takes place.*

In [15, 16, 22, 23, 24, 26] it has been shown that we are indeed able to recover the probabilities that appear in quantum theory by considering every measurement as a collection of so called 'hidden measurements' on which we introduce a lack of knowledge through a so called mathematical *weight*-representative for the relative frequency of occurrence of the hidden measurements when the original measurement actually takes place. These ideas have been further developed on a formal level within an abstract mathematical setup called the *hidden measurement formalism* for physical measurements [15, 16, 22, 23, 24, 26, 27]. For an overview of other applications of the *hidden measurement approach* we refer to [18, 19, 20, 21, 24, 25, 28, 29, 30, 31, 32, 33].

In this paper we proceed as follows. First we briefly describe a classical physical entity, and we explain the meaning of probabilities within the framework of classical physics. Secondly, we explain in which way quantum entities and quantum probabilities differ from such classical ones. In the following section we introduce *aspects of creation*, and we show how they fit in our traditional picture. Finally we present a model within macroscopic physical reality that generates exactly the quantum probabilities, and in which appear only probabilities that correspond with a lack of knowledge.

2 Probabilities in classical systems

In this section we introduce the notion of probability, and how it occurs in classical physics. In classical physics we describe classical systems. A classical system can be conceived as an entity that represents a well-defined part of reality, and which is separated from its environment. A second important aspect that characterizes a classical physical entity is that we are in principle able to attach definite values *true* or *false* to all the physical quantities necessary and sufficient to give a full representation of our physical system. We explain this in more detail. As a starting point we consider the *elements of physical reality*, introduced by Einstein, Podolsky and Rosen who said in their 1935-paper [2]: 'If, without in any way disturbing a system, we can predict with certainty the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity'. We now quote Piron [14] to explain what we mean by an *actual property*[1]: 'A property is actual (or the system possesses an 'element of reality') if it is 'certain' that the positive result would be obtained if the experimentator would decide to perform the corresponding experiment', where it is presupposed that every property that one can attach to a physical system is testable, i.e., there exists an experiment that verifies it. Since the set of all possible physical quantities which *could* describe our entity are completely characterized by the set of all properties of the entity (actual, or not, which is called *potential* [9, 11, 34]), we are for every classical entity able to attach a value *true* or *false* to every property respectively corresponding with the property being actual or potential[2]. Of course, in certain situations we might not know exactly the value of every property of the physical entity (although they really exist). This means in fact that *we have* a lack of knowledge on the properties of the entity. The mathematical tool delivered to deal with such a situation of a lack of knowledge on a classical entity is a *probability measure* [1, 3]. In such a probability representation, if something is *true to our knowledge* it requires that we attach a value 1 to it, and if it is *false to our knowledge* it requires a value 0. In a lack of knowledge situation, we are dealing with values lying between (and not equal to) 0 and 1, representative for a lack of certainty. The most important example where we encounter such a lack of knowledge situation is the one of a statistical ensemble, i.e., a collection of entities to which we attach only global parameters (for example temperature, density, velocity distributions etc ...). Consider for example a quantity of a liquid within a barrel: For every individual molecule in this liquid, we know that it is located within the barrel, but we don't know where exactly in the barrel; as a consequence, the properties related to the individual molecules are not exactly known; nonetheless, we do know which

[1] More details on this specific notion of a property can be found in [9, 11, 34].

[2] At this point, the present reader might get a feeling of conceptual overkill. Therefore, we already mention that it will be here that the troubles start in the quantum case: false will not be implied by potential. As such, this more evolved conceptual picture will enable us to point at the differences between the classical and quantum system in a scheme that hosts both of them [9, 11].

ones are very probable and which aren't.

3 Probabilities in quantum systems

For more than sixty years now, scientists have been trying to really understand quantum mechanics. This long period is at least partly due to the fact that we are very attached to the classical picture of nature. According to Einstein and many others — who believed in what we can call the classical picture of nature — the quantum description which should represent an entity is incomplete. This has been a point of discussion for many years, and we have now come to the conclusion that we believe that it is indeed possible to give a complete realistic description of a quantum entity. But what was the cause for this so called incompleteness? The fact is that when the formalism of quantum mechanics had been constructed around 1927 to describe microscopic systems, the only mathematical expression to describe an entity was an expression about possibilities or tendencies. In most cases where we use the standard quantum formalism, we can't give definite values to all physical quantities which are necessary and sufficient to give a full representation of our physical system. The probability assignments are the most the theory can say about a quantum system we want to observe or to measure. In fact, one can prove that for every possible preparation we could make when we want to check if an entity has a well defined property, there are always other properties about which we cannot be certain. Referring to the previous section, this means that not all potential properties are false. When we perform an experiment that tests them, both answers might occur, namely false and true. In fact, only very few properties that are not actual will be false. When we use the standard quantum formalism, one has to accept that for these properties that are not true and not false, there *exist* probabilities that do not describe just a lack of knowledge about the exact situation of our entity, but that are a priori linked with the quantum world. That's why one also calls them objective probabilities, contrary to the classical probabilities that refer to an incomplete knowledge of a given situation. These classical probabilities, which are the same for every human being who wants to observe an entity, correspond in fact to a lack of knowledge of the 'state' of the entity.[3] Until now, one has not been able to give

[3]We consider the quantum state to give us an as complete as possible representation of the elements of reality of the entity under consideration. With the words of C. Piron [14]:'The state is nothing else than the collection of all actual properties of the given system.'

a real content to this picture of objective (ontological) probabilities because they imply that the existence of an entity can be asserted only with a certain level of probability. In this article we want to explain another kind of view which doesn't need objective probabilities. This view, which we have called the hidden measurement approach, assumes that quantum probabilities are caused by a lack of knowledge such as we are used to from the classical case. Here, quantum probabilities aren't objective, but the kind of lack of knowledge differs from the one in classical physics in the sense that a lack of knowledge on the state of the system gives rise to a classical probability model, and a lack of knowledge concerning the measuring process itself introduces a non-classical model. The presence of the quantum probability is due to the fact that before the measurement process starts, it is not determined which of the hidden measurement processes shall take place. To explain what we really mean here, we shall first have to take a closer look at the aspects of creation which are inherent to measurements that change the state.

4 Aspects of creation

Within the hidden measurement approach of quantum mechanics we are able to explain the appearances of quantum-like probabilities. Measuring or observing a physical reality involves to some extent in the quantum case, also the creation of the reality observed. What we mean is that some of the elements of physical reality are being created in the course of measurement. The act of creation that sometimes takes place during a measurement is something we are familiar with from our ordinary macroscopic world. The fact that it only now attracts our attention is that in the quantum case, measurements that in ordinary reality are just observations also seem to contain a creative element in the quantum case. We will give an example, which has already been put forward in [20], and which will explain the 'creation-discovery'-view a bit more. Consider a survey to determine the opinion on nuclear energy of an arbitrary selected group persons. A few interviewers have to find out whether one is 'for' or 'against' the use of nuclear energy. The aim is to obtain the true opinion of the members of the survey, therefore the interviewers have been provided with full information about the problem. Before an interviewer asks the question to find out if one is 'for' or 'against' nuclear energy, a lot of the members of our survey group will already have a strict opinion (for or against) on the matter, while other members haven't thought the subject over in advance and so haven't made their mind up yet. By asking the question the interviewer will, from those members who had a strict opinion in advance, 'discover' if they are for or against nuclear energy. The ones who didn't have an opinion in advance, so the ones who are in doubt about the subject when the question is being asked, will receive enough information so that also they can give a 'for or against' answer. We see here that by this experiment persons are pulled out a 'doubtful'-state into a 'for or against'-state. This change of state is caused by the creation-aspect that happens during the measurement or survey. How the change of state will evolve in the 'doubtful-person'-case, depends for a large part on the persuasive force of the interviewer and on the information he offers. In other words : we say that the change of state depends in this 'doubtful-person'-case on the influence of the measurement context. When we are only dealing with a discovery, there's almost no influence of the measurement context. Let's now look at the probabilities. When an interviewer meets a member of the survey group, he does not know his/her opinion and without explicitly asking the question, he can only make a probabilistic prediction. We make the following assumption on the probabilities characteristic of the interaction with the interviewer and the doubtful person: an interviewer meeting such a member of the survey has a 40 % chance of finding a person who will be for nuclear energy and a 60 % chance of finding one against (here, we attribute probabilities to the change of state of persons). Suppose that after we have asked the question, of all our 1000 members we will find 400 for and 600 against. Possible initial distributions of the members of the survey which yield this result could be: (i) in advance, 400 are for and 600 are against, (ii) in advance, all 1000 have no definite opinion, (iii) n are for (we choose n between 0 and 400), $\frac{3}{2}n$ are against, and 1000-$\frac{5}{2}n$ have no opinion. We want to stress the fact that the result of the survey depends both on the initial distribution of the members of the survey and on the assumption we made on the interaction of the interviewer with a doubtful person. We will now analyse this in more detail. Although all above examples yield the same result, this result emerged from situations with a different nature. In (i) there was no act of creation by the interviewer since all members of the survey had a definite answer in advance. Thus this situation corresponds to a classical situation (compare to 'all properties are true or false' in section 2). In all the other situations the result is partially due to our probabilistic assumption of the interviewer creating an answer 'for or against' during his interaction with a member of the survey group. The probabilities attributed to the change of state of a doubtful person, can not be attributed to a lack of knowledge of a more complete specification of the real state of the person, because a more complete specification simply does not exist. The probabilities about the state in which the other persons with a fixed opinion will end up, can indeed be attributed to a lack of knowledge of a more complete specification of the real state of the person. As we will point out in the next paragraph, the creation-discovery picture gives a very natural explanation for the existence of quantum probabilities. Quantum probabilities are attributed to the lack of knowledge concerning the measurement process and not to the lack of knowledge on the state of the system.

5 The quantum machine

We'll go more into detail to justify our creation-discovery picture for the quantum case. The way to do this is to talk about what we call the quantum machine[4]. This machine provides a model that is applicable to a spin-$\frac{1}{2}$ quantum entity. The 'lack of knowledge' on the measurement process will be presented by the parameter $\lambda \in [0,1]$. The entity in Fig. 1 is a point particle that moves on the surface of a sphere with center 0 and radius 1. If the point has coordinates v, we denote the state corresponding to this point as p_v. A measurement e_u on the entity in a state p_v is defined in the following way: Consider a straight line segment with one of its endpoints in the point u of the sphere, and the other one in the antipodal point $-u$. We'll denote this segment by $[-u,u]$. We project v orthogonally on $[-u,u]$ and obtain the point v'. This point defines two segments $[-u,v']$ and $[v',u]$ (see Fig.1).

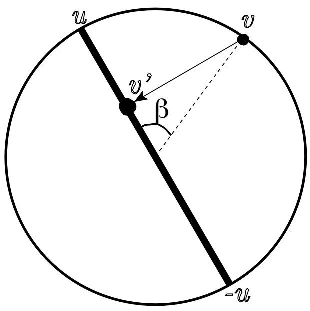

Figure 1: Illustration of a measurement e_u on our model for a spin-$\frac{1}{2}$ entity when the initial state is p_v.

[4]A variant of this model has first been introduced in [15, 16]. A similar modelization is also applied in [12].

Consider now a random variable λ defined on the segment $[-u, u]$, and suppose that the relative frequency of appearance of the possible λ is uniformly distributed on $[-u, u]$. If $\lambda \in [-u, v']$, the point corresponding to the state of the entity moves to u along $[v', u]$ and we obtain a state p_u. If $\lambda \in]v', u]$, the point moves to $-u$ along $[v', -u]$ and we obtain p_{-u}. As a consequence, there are two possible outcome states for this measurement e_u: p_u and p_{-u}. From a more mechanical point of view, the segment $[-u, u]$ can be seen as a uniform 'elastic', that can break in every one of its points in $[-u, u]$ with the same probability. Before we put the machine on, there is no way to find out in which point the elastic will break. In the case we push the start-button of the quantum machine, we get the following image: the point particle falls from its original location on the elastic such that its 'falling-direction' is orthogonal to the elastic, and sticks to it. Then the elastic breaks and the particle is torn to one of the two original endpoints of the elastic. If we decide to perform a measurement in the u direction when we start with a particle located in a point v on the unit sphere, it is easy to calculate probabilities for a transition of the particle to u or $-u$:

$$P[v \to u] = \frac{1 + cos\beta}{2} = cos^2\frac{\beta}{2} = 1 - P[v \to -u]$$

where β is the angle between the vectors u and v. These probabilities are the same as one obtains in a Stern-Gerlach measurement on spin-$\frac{1}{2}$ quantum entities, i.e., this model system generates quantum probabilities. For more details on this model system we refer to [15, 16, 18, 21, 22, 25, 29, 30, 32]. A similar model for a quantum entity described in a three dimensional Hilbert space can be found in [28]. At every moment it is possible to look at what happens within this machine. A classical machine will be one where we install a rope which has the ability only to break in one point, such that we get classical probabilities which are in principle for a well defined initial location of the point particle only 1 or 0 (we omit the neglectable situation of the point-particle falling on the breaking point, since this is an event with a so called *zero measure*). Given the quantum machine, we are now able to visualize an emergence of the quantum probabilities. As we mentioned before, the probability appears due to a lack of knowledge about the measurement process. In our quantum machine this lack of knowledge is exactly referring to the point where the elastic breaks. We don't have any means to find out where the elastic will break when we set our quantum machine in action. The quantum probabilities might be looked at in this way. We mean that our quantum probabilities are calculated in a way in which we take the probability over different classical machines, put on action. Each of the different classical machines considered has a rope of the same length as the big quantum machine's elastic, but now the rope is only breakable in one point. All these classical machines differ in the sense that their ropes will break in another point if set in action. We need to consider as many classical machines as the quantum-elastic has breaking points. Now stated shortly, a quantum machine in action is the same as one of all the considered classical machines in action. The only point is that we don't know which classical machine with its specific breaking point-rope will actually be put into action. This special classical machine is our hidden measurement. Although this machine may be hidden, it influences the result of the measurement in such a way that it is responsible for the creative aspect.

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