Quantum Structures and their Future Importance*

Diederik Aerts

FUND and CLEA,
Brussels Free University,
Pleinlaan 2, 1050 Brussels;
e-mail: diraerts@vub.ac.be

Relativity theory, formulated in great part by one person, Albert Einstein [1], is founded on the concept of 'event', which is a concept that is physically well defined and understood [2]. Within relativity theory itself, the events are represented by the points of a four dimensional space-time continuum. In this way, relativity theory has a well defined physical and mathematical base.

The development of quantum mechanics proceeded in a rather haphazard manner, with the introduction of many ill-defined and poorly understood new concepts. During its first years, from 1890 to 1925, quantum mechanics, commonly referred to as the *'old quantum theory'*, did not even possess a coherent mathematical basis. In 1926 Werner Heisenberg formulated matrix mechanics in a mainly technical effort to explain and describe the energy spectrum of the atoms [3]. Around the same time Erwin Schrödinger elaborated wave mechanics which seemed to have a more solid physical base: a general idea of wave-particle duality, in the spirit of Louis de Broglie or Niels Bohr [4]. But then Paul Adrien Maurice Dirac [5] and later John Von Neumann [6] proved that the matrix mechanics of Heisenberg and the wave mechanics of Schrödinger are equivalent: they can be constructed as two mathematical representations of one and the same vector space, the Hilbert space. This fundamental result indicated already that the 'de Broglie wave' and the 'Bohr wave' are not physical waves and that the state of a quantum entity is an abstract concept: a vector in an abstract vector space.

The abstract Hilbert space formulation of quantum mechanics, as introduced by John von Neumann in 1932 [6], and now commonly referred to as 'Standard Quantum Mechanics' (SQM), uses an elaborate and sophisticated mathematical formalism, of which the basic concepts remain however vague and unclear from

*Published as: Aerts, D., 2001, "Quantum Structures and their future importance", *Soft Computing*, **5**, 131.

the physical point of view. The predictive success of the theory was however so remarkable that it immediately was accepted as constituting a fundamental contribution to physics. The problems surrounding its conceptual basis however led to a broad and prolonged debate in which all the leading physicists of the time participated [7].

The study of the structure of quantum mechanics is almost as old as quantum mechanics itself. The fact that the two early versions of quantum mechanics - the matrix mechanics of Werner Heisenberg and the wave mechanics of Erwin Schrödinger - were shown to be structurally equivalent, made it already clear in the early days that the study of the structure itself would be very important. The study of quantum structures had two main goals: (1) the elaboration of SQM with a physical and mathematical base that is as clear as the one that exists in relativity theory, and (2) the axiomatic foundations of SQM with a set of physically plausible and mathematically well defined axioms.

In 1936, Garret Birkhoff and John Von Neumann, wrote an article entitled 'The logic of quantum mechanics'. They show that if one introduces the concept of 'operational proposition' and its representation in SQM by an orthogonal projection operator of the Hilbert space, it can be shown that the set of the 'experimental propositions' does not form a Boolean algebra, as it is the case for the set of propositions of classical logic [8]. As a consequence of this article the field called 'quantum logic' came into existence: an investigation on the logic of SQM.

An interesting idea was brought forward. Relativity theory is a theory based on the concept of 'event' and a mathematical structure of a four dimensional space-time continuum. This space-time continuum contains a non Euclidean geometry. Could it be that the article of Birkhoff and Von Neumann indicates that quantum mechanics should be based on a non Boolean logic in the same sense as relativity theory is based on a non Euclidean geometry? This is a fascinating idea, because if quantum mechanics were based on a non Boolean logic, this would perhaps explain why paradoxes are so abundant in quantum mechanics: the paradoxes would then arise because classical Boolean logic is used to analyze a situation that intrinsically incorporates a non classical, non Boolean logic.

Following this idea quantum logic was developed as a new logic and also as a detailed study of the logico-algebraic structures that are contained in the mathematical apparatus of quantum mechanics. The systematic study of the logico-algebraic structures related to quantum mechanics was very fruitful and we refer to [9] [10] for a good overview. On the philosophical question of whether quantum logic constitutes a fundamental new logic for nature a debate started. An account of this discussion can be found in [7].

We want to put forward our opinion about this matter and explain why the word 'quantum logic' was not the best word to choose to indicate the scientific activity that has been taking place within this field. If 'logic' is the formalization of the 'process of our reflection', then quantum logic is not a new logic. Indeed, we reflect following the same formal rules whether we reflect about classical parts of reality or whether we reflect about quantum parts of reality. Birkhoff and Von Neumann, when they wrote their article in 1936, were already aware of this, and that is why they introduced the concept of 'experimental proposition'. It could indeed be that, even if we reason within the same formal structure about quantum entities as we do about classical entities, the structure of the 'experimental propositions' that we can use are different in both cases. With experimental proposition is meant a proposition that is connected in a well defined way with an experiment that can test this proposition. And indeed, the set of experimental propositions connected to a quantum entity has a different structure than the set of experimental propositions connected to a classical entity. We believe however that this difference in structure of the sets of experimental propositions is only a little piece of the problem, and even not the most important one. We can show for example that even the set of experimental propositions of a macroscopic entity does not necessarily have the structure of a Boolean algebra. This means that the only fact of limiting oneself to the description of the set of 'experimental' propositions already brings us out of the category of Boolean structures, whether the studied entities are microscopic or macroscopic [11].

It is our opinion that the difference between the logico-algebraic structures connected to a quantum entity and the logico-algebraic structures connected to a classical entity is due to the fact that the structures of our 'possibilities of active experimenting' with these entities is different. Not only the logical aspects of these possibilities of active experimenting but the profound nature of these possibilities of active experimenting is different. And this is not a subjective matter due to, for example, our incapacity of experimenting actively in the same way with a quantum entity as with a classical entity. It is the profound difference in nature of the quantum entity that is at the origin of the fact that the structure of our possibilities of active experimenting with this entity is different. We will not explain in great generality what we mean with this statement in this paper and we refer the reader to [12] for such a presentation. We only mention this analysis because it is important for the point that we want to make in this paper about the future importance of quantum structure research.

Contemporary technology strives for a fine mastership of complex phenomena. As it tries to assume a precise control of the use of energy, of the memorization and transmission of information, of the creation of new materials and of biological and medical practice, a need for more and more miniaturization and rapidity is unavoidable. There will be a constant attempt to control and manipulate energy and information on the micro level and during shorter lapses of time. This means that the future technology will definitely and inescapably be a quantum technology. This development will call for the introduction of clear, well understood concepts within the quantum domain. Theories that directly describe the possibilities of active experimenting and lend the theoretical concepts that are necessary to formulate these possibilities are superior to theories that only contain mathematical concepts that are even not very well understood. This is the reason that we belief that the theoretical studies and elaborations that nowadays are taking place within the field of quantum structure research will be used and become increasingly important in the future.

To illustrate this general thought we would like to mention a problem that appears within the new field of quantum computing. It is stated in many papers on quantum computing that nowadays only single bit quantum computers can built in the laboratory because of the well known effect of decoherence. This statement is carried by the intuitive image that quantum nature is coherent and can only be kept when shielded off from the perturbations of the environment, and that is are these perturbations of the environment that destroy this quantum coherence. This is not correct. If one analysis SQM from a more fundamental point of view, it can be shown that the decoherence finds its origin in the non-local effects that are always present in a quantum entity. These non-local effects make that the quantum entity does not remain localized and spreads out in itself. Hence it is in fact that other way around: the problem to built a genuine quantum computer is rooted in the fact that a quantum entity cannot be shielded i such a way that it would not spread out its quantum calculations outside of the original quantum computer environment, and in this sense get lost. This means that the practical struggle of genuine quantum computing will not be a matter of technically isolating the quantum computer, but will touch at the deep problem of non-locality. In the field of quantum structure research this problem has been investigated in its most understandable form and many possibilities and conclusions, not exciting in SQM, have been brought forward. Since the author himself - this explains the nature of the example - has been working during years on this problem, we refer to [13] [14] [15] [16] [17], were it can be seen in which way the concept of non-locality, is tried to be unraveled and connected with real laboratory situations.

Acknowledgement: The research for this paper was realized with the aid of AWI-grant Caw96/54a of the Flemish Community. Diederik Aerts is senior research associate of the Fund for Scientific Research Flanders.

References

[1] Einstein, A., (1905). Zur elektrodynamik bewegter Körper, *Ann. Phys.*, **17**, 891.

[2] Misner, C. W., Thorne, K. S. and Wheeler J. A., (1970). *Gravitation*, Freeman and Company, San Francisco.

[3] Heisenberg, W., (1925). Uber quantentheoretische Umdeuting kinematischer und mechanischer Beziehungen, *Zeitschrift für Physik*, **33**, 879.

[4] Schrödinger, E., (1926). Quantisiering als Eigenwertproblem, *Ann. der Phys.*, **79**, 361.

[5] Dirac, P.A.M., (1930). *The Principles of Quantum Mechanics*, Oxford University Press.

[6] von Neumann, J., (1932). *Mathematische Grundlagen der Quantummechanik*, Berlin, Springer.

[7] Jammer, M., (1974). *The Philosophy of Quantum Mechanics*, Wiley and Sons, New York, Sydney, Toronto.

[8] Birkhoff, G. and Von Neumann, J., (1936). The logic of quantum mechanics, *Annals of Mathematics*, **37**, 823.

[9] Beltrametti, E. and Cassinelli, G., (1981). *The logic of quantum mechanics*, Addison-Wesley, Reading, Mass.

[10] Aerts, D. and Pykacz, J., eds., (1999). *Quantum Structures and the Nature of Reality*, the *Indigo Book of Einstein meets Magritte*, Kluwer Academic, Dordrecht.

[11] Aerts, D. and Durt, T., (1994). Quantum. Classical and Intermediate, an illustrative example, *Found. Phys.* **24**, 1353.

[12] Aerts, D., (1999). Foundations of quantum physics: a general realistic and operational approach, *Int. J. Theor. Phys.*, **38**, 289.

[13] Aerts, D., (1982). Description of many physical entities without the paradoxes encountered in quantum mechanics, *Found. Phys.*, **12**, 1131.

[14] Aerts, D., (1988). The description of separated systems and quantum mechanics and a possible explanation for the probabilities of quantum mechanics, in *Micro-physical Reality and Quantum Formalism*, eds. A. van der Merwe et al., Kluwer Academic Publishers, 97.

[15] Aerts, D. and Reignier, J., (1991). On the problem of non-locality in quantum mechanics, *Helv. Phys. Acta*, **64**, 527.

[16] Aerts, D., (1994). Quantum structures, separated physical entities and probability, *Found. Phys.*, **24**,1227.

[17] Aerts, D., (1999). The stuff the world is made of: physics and reality, in *Einstein meets Magritte: an interdisciplinary reflection*, eds. D. Aerts, J. Broekaert and E. Mathijs, Kluwer Academic, Dordrecht.