

Overall, in this case,

$$\begin{aligned} & \text{Prob}(\text{happy, unknown employment}) \\ &= |M \cdot U(t) \cdot |\psi_{\text{employed}}\rangle|^2 \cdot \text{Prob}(\text{employed}) \\ &+ |M \cdot U(t) \cdot |\psi_{\sim \text{employed}}\rangle|^2 \cdot \text{Prob}(\sim \text{employed}). \end{aligned}$$

It should be clear that in such a case there are no interference terms and the quantum result converges to the classical one. Note that the “quantum” reasoner is still uncertain about whether she will be employed. The crucial difference is that in this case she knows she will resolve the uncertainty regarding employment, before her inner reflection. Therefore, regardless of the outcome regarding employment, the evolved state will be a state that is not a superposition one.

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Open Peer Commentary

Quantum structure and human thought

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Abstract: We support the authors’ claims, except that we point out that also quantum structure different from quantum probability abundantly plays a role in human cognition. We put forward several elements to illustrate our point, mentioning entanglement, contextuality, interference, and emergence as effects, and states, observables, complex numbers, and Fock space as specific mathematical structures.

The authors convincingly demonstrate the greater potential of quantum probability as compared with classical probability in modeling situations of human cognition, giving various examples to illustrate their analysis. In our commentary, we provide additional arguments to support their claim and approach. We want to point out, however, that it is not just quantum probability, but also much more specific quantum structures, quantum states, observables, complex numbers, and typical quantum spaces – for example, Fock space – that on a deep level provide a modeling of the structure of human thought itself.

A first insight about quantum structure in human cognition came to us with the characterizations of classical and quantum probability following from the hidden-variable investigation in quantum theory – that is, the question of whether classical probability can model the experimental data of quantum theory (Bell 1964; Einstein et al. 1935). From these investigations, it follows that when probability is applied generally to a physical system, classical probability models the lack of knowledge of an underlying deterministic reality, whereas non-classical probability, and possibly quantum probability, results when indeterminism arises from the interaction between (measurement) context and system, introducing genuine potentiality for the system states (Aerts 1986). This allowed the identification of situations in macroscopic reality entailing such non-classical indeterminism and therefore being unable to be modeled by classical probability (Aerts 1986; Aerts et al. 1993). It shows that opinion polls, where human decisions are intrinsically influenced by the context, constitute such situations, and therefore entail non-classical probability (Aerts & Aerts 1995). Our first argument to support and strengthen the authors’ claim is that a generalization of classical probability is necessary whenever intrinsically contextual situations evoking indeterminism and potentiality are present (Aerts 1986). We believe this to be commonly the case in situations of human cognition, and believe quantum probability to be a plausible description for this indeterminism and potentiality.

Another result followed from studying the structure and dynamics of human concepts themselves: how concepts combine to form sentences and carry meaning in human thought. An investigation into the relation of concepts to their exemplars allowed for the devising of a *Gedankenexperiment* violating Bell’s inequalities, identifying the presence of quantum entanglement (Aerts et al. 2000). Considering a combination of concepts and its relation to exemplars led to an experimental violation of Bell’s inequalities, proving that concepts entangle when they combine (Aerts & Sozzo 2011a). We studied the guppy effect: Whereas a guppy is considered a very typical “pet-fish,” it is regarded as neither a typical “pet” nor a typical “fish.” The study of this effect, proved to be impossible to model with classical fuzzy sets by Osherson and Smith (1981), led us to develop a quantum-based concept theory presenting the guppy effect as a non-classical context influence. Concepts are modeled as entities in states in a complex Hilbert space, and quantities such as typicality are described by observables in the quantum formalism (Aerts & Gabora 2005a; 2005b; Gabora & Aerts 2002). Our second argument to support and strengthen the authors’ approach is that next to quantum effects such as entanglement and contextuality, typical quantum representations of states and observables appear in the combination dynamics of human concepts.

An abundance of experimental data violating set theoretical and classical logic relations in the study of the conjunctions and disjunctions of concepts (Hampton 1988b; 1998b) led to the identification of new quantum effects – interference and emergence – when these data were modeled using our quantum concept formalism. Fock space, a special Hilbert space also used in quantum field theory, turns out to constitute a natural environment for these data. For the combination of two concepts, the first sector of Fock space, mathematically describing interference, corresponds to emergent aspects of human thought, and the second sector, mathematically describing entanglement, corresponds to logical aspects of human thought (Aerts 2007; 2009; 2011; Aerts & D’Hooghe 2009; Aerts et al., in press). The quantum superposition in Fock space, representing both emergent and logical thought, models Hampton’s (1988a; 1988b) data well in our approach. Our third argument to support and strengthen the authors’ analysis is that the quantum formalism, and many detailed elements of its mathematical structure, for example, Fock space, has proved to be relevant for the structure of human thought itself.

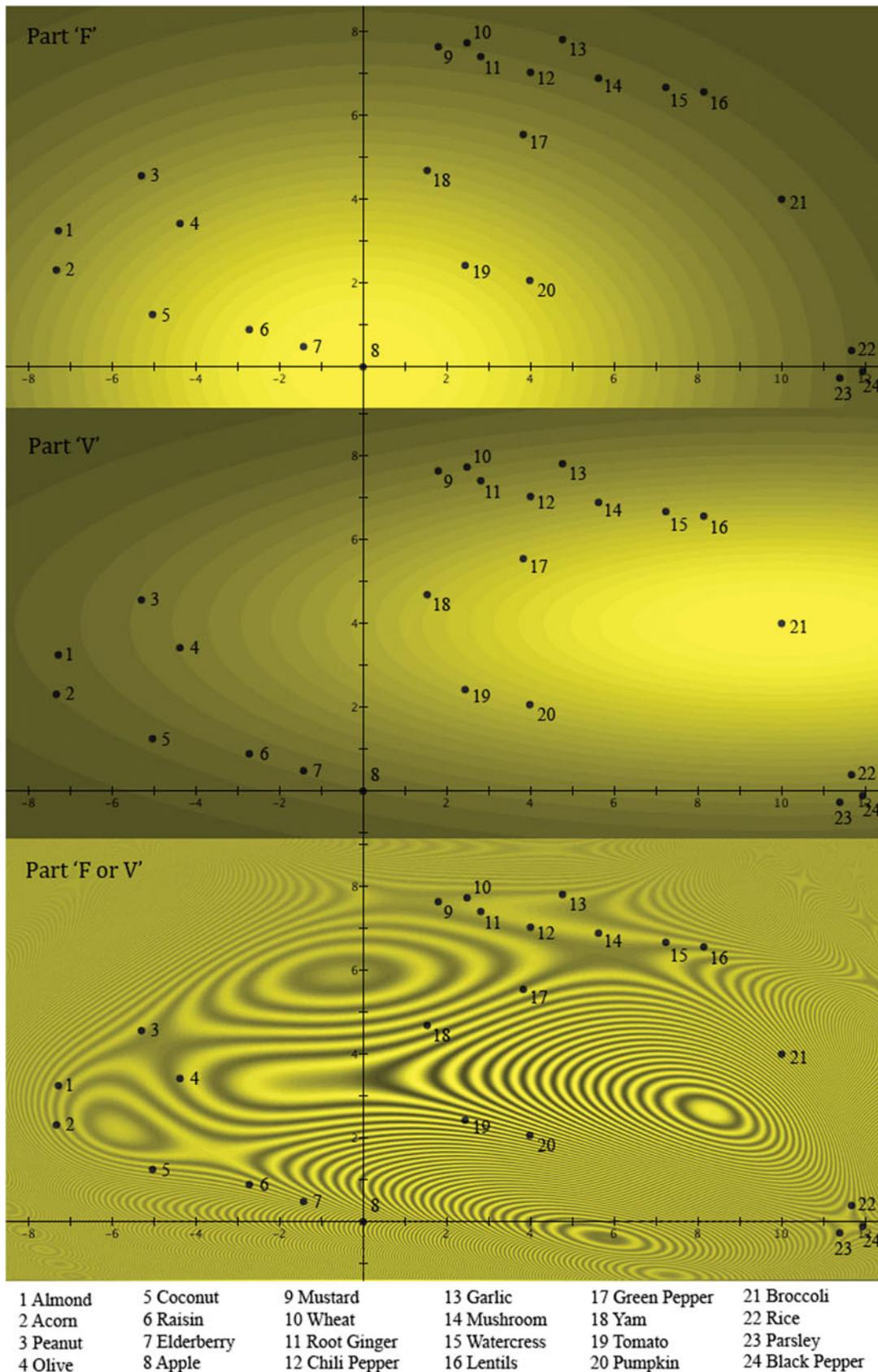


Figure 1 (Aerts et al.). Part “F,” Part “V,” and Part “F or V” are a graphical representation of the relative membership weights of the indicated exemplars with respect to “fruits,” “vegetables,” and “fruits or vegetables,” respectively. The light intensity at the spots where the exemplars are located is a measure of the relative membership weight at that spot, and hence the graphs can be interpreted as light sources passing through slits “F,” “V,” and “F and V.”

Table 1 (Aerts et al.). *Relative membership weights of exemplars with respect to fruits, vegetables and fruits or vegetables as measured by Hampton (1988a)*

		Fruits	Vegetables	Fruits or vegetables
1	Almond	0.0359	0.0133	0.0269
2	Acorn	0.0425	0.0108	0.0249
3	Peanut	0.0372	0.0220	0.0269
4	Olive	0.0586	0.0269	0.0415
5	Coconut	0.0755	0.0125	0.0604
6	Raisin	0.1026	0.0170	0.0555
7	Elderberry	0.1138	0.0170	0.0480
8	Apple	0.1184	0.0688	0.0670
9	Mustard	0.0149	0.0250	0.0146
10	Wheat	0.0138	0.0255	0.0165
11	Root ginger	0.0157	0.0323	0.0385
12	Chili pepper	0.0167	0.0446	0.0323
13	Garlic	0.0100	0.0301	0.0293
14	Mushroom	0.0140	0.0545	0.0604
15	Watercress	0.0112	0.0658	0.0482
16	Lentils	0.0095	0.0713	0.0338
17	Green pepper	0.0324	0.0788	0.0506
18	Yam	0.0533	0.0724	0.0541
19	Tomato	0.0881	0.0679	0.0688
20	Pumpkin	0.0797	0.0713	0.0579
21	Broccoli	0.0143	0.1284	0.0642
22	Rice	0.0140	0.0412	0.0248
23	Parsley	0.0155	0.0266	0.0308
24	Black pepper	0.0127	0.0294	0.0222

We finish our commentary by presenting a graphic illustration of the interference of concepts as it appears in our quantum concept theory. Figure 1 represents the cognitive interference of the two concepts “fruits” and “vegetables” combined in the disjunction “fruits or vegetables.” Part “F,” Part “V,” and Part “F or V” illustrate the relative membership weights of the different exemplars with respect to “fruits,” “vegetables,” and “fruits or vegetables,” respectively, measured in Hampton (1988a) and presented in Table 1. The illustration is built following standard quantum theory in a Hilbert space of complex wave functions in a plane. The exemplars are located at spots of the plane such that the squares of the absolute values of the quantum wave functions for “fruits,” “vegetables” and “fruits or vegetables” coincide with the relative membership weights measured.

The wave function for “fruits or vegetables” is the normalized sum – that is, the superposition – of the two wave functions for “fruits” and for “vegetables,” and hence the square of its absolute value includes an interference term. The light intensity at the spots where the exemplars are located is a measure of the relative membership weight at that spot, which means that the graphs can be seen as representations of light passing through slits, where Part “F” corresponds to slit “F” open, Part “V” to slit “V” open, and Part “F or V” to both slits “F” and “V” open. Hence, the graphs illustrate the cognitive interference of “fruits or vegetables” in analogy with the famous double-slit interference pattern of quantum theory (Feynman 1988). The interference pattern is clearly visible (Part “F or V” of Fig. 1), and very similar to well-known interference patterns of light passing through an elastic material under stress. Mathematical details can be found in Aerts et al. (in press).

At home in the quantum world

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Abstract: One among many misleading quotations about the alleged mysteries of quantum theory is from Feynman (1965): “I think I can safely say that nobody understands quantum mechanics.” Today we know that quantum theory describes many aspects of our world in a fully intelligible fashion. Pothos & Busemeyer (P&B) propose ways in which this may include psychology and cognitive science.

It was an old idea by Niels Bohr, one of the founding architects of quantum physics, that central features of quantum theory, such as complementarity, are also of pivotal significance beyond the domain of physics. Bohr became familiar with the notion of complementarity through the psychologist Edgar Rubin and, indirectly, William James (Holton 1970). Although Bohr always insisted on the extraphysical relevance of complementarity, he never elaborated this idea in concrete detail, and for a long time no one else did so either. This situation has changed; there are now a number of research programs applying key notions of quantum theory beyond physics, in particular to psychology and cognitive science.

The first steps in this direction were made by Aerts and collaborators in the early 1990s (Aerts & Aerts 1995) in the framework of non-Boolean logic of incompatible (complementary) propositions. Alternative ideas come from Khremnikov (1999), focusing on non-classical probabilities, and Atmanspacher et al. (2002), proposing an algebraic framework with non-commuting operations. More recently, Bruza and colleagues as well as Busemeyer and colleagues have moved this novel field of research even more into the center of attention. The target article by Pothos & Busemeyer (P&B), and a novel monograph by Busemeyer and Bruza (2012), reflect these developments.

Intuitively, it is plausible that non-commuting operations or non-Boolean logic should be relevant, even inevitable, for mental systems. The non-commutativity of operations simply means that the sequence in which operations are applied matters for the final result. This is so if individual mental states are assumed to be dispersive (as individual quantum states are, as opposed to classical states). As a consequence, their observation amounts not only to registering a value, but entails a backreaction changing the observed state: something that seems evident for mental systems.

Non-Boolean logic refers to propositions that may have unsharp truth values beyond “yes” or “no.” However, this is not the result of subjective ignorance but must be understood as an intrinsic feature. The proper framework for a logic of incompatible propositions is a partial Boolean lattice (Primas 2007), where locally Boolean sublattices are pasted together in a globally non-Boolean way – just like an algebra of generally non-commuting operations may contain a subset of commuting operations.

Although these formal extensions are essential for quantum theory, they have no dramatic effect on the way in which experiments are evaluated. The reason is that the measuring tools, even in quantum physics, are typically Boolean filters, and, therefore, virtually all textbooks of quantum physics get along with standard probability theory à la Kolmogorov. Only if incompatible experimental scenarios are to be comprehensively discussed in one single picture do the peculiarities provided by non-classical thinking become evident and force us to leave outmoded classical reasoning.

In this sense, the authors use the notion of “quantum probability” for psychological and cognitive models and their predictions (cf. Gudder 1988; Redei & Summers 2007). As Busemeyer points out, an experiment in psychology is defined as a collection of experimental conditions. Each one of them produces indivisible outcomes

forming a complete set of mutually exclusive events. Whereas Kolmogorov probabilities refer to events for a single condition, quantum probabilities refer to the entire set of incompatible conditions, necessary for a comprehensive description of the experiment.

In such a description, events are represented as subspaces of a Hilbert space (as in quantum physics), and all subspaces correspond to orthogonal projectors. A state is defined as a vector, and the probability of an event equals the squared length of the projection of the state onto the subspace representing that event. As all events under each single experimental condition commute, they form a Boolean algebra and the probabilities assigned to them satisfy the axiomatics of Kolmogorov. However, all events of the entire experiment (i.e., the events of all experimental conditions) only form a partial Boolean algebra if some of them do not commute. And as Kolmogorov's axioms imply Bayes' rule, Bayesian reasoning, very influential in psychology, will generally fail to describe experiments with incompatible conditions properly.

Whereas the authors focus on decision theory, routes to be explored in more detail include uncertainty relations, in which order effects arise in variances in addition to mean shifts (Atmanspacher & Römer 2012). A key feature of quantum theory, entanglement as tested by Bell-type inequalities, has been suggested by Atmanspacher and Filk (2010) for bistable perception and by Bruza et al. (2012) for non-decomposable concept combinations.

Another possible move to incorporate complementarity and entanglement in psychology is based on a state space description of mental systems. If mental states are defined on the basis of cells of a state space partition, then this partition needs to be well tailored to lead to robustly defined states. Ad hoc chosen partitions will generally create incompatible descriptions (Atmanspacher & beim Graben 2007) and states may become entangled (beim Graben et al. 2013). This way it is possible to understand why mental activity may exhibit features of quantum behavior whereas the underlying neural dynamics are strictly classical.

A further important issue is the complexity or parsimony of Hilbert space models as compared with classical (Bayesian, Markov) models. Atmanspacher and Römer (2012) proposed an option to test limitations of Hilbert space modeling by outcomes of particular joint measurements. Such tests presuppose that the situation under study is framed well enough to enable well-defined research questions; a requirement that must be carefully observed to avoid superficial reasoning without sustainable substance.

With the necessary caution, I am optimistic that this novel field will grow from work in progress to an important subject area of psychology. A quantum theoretically inspired understanding of reality, including cognition, will force us to revise plugged-in clichés of thinking and resist overly naive world views. The Boolean “either-or” in logic and the law of commutativity in elementary calculations are special cases with their own significance, but it would be wrong to think that their generalization holds potential only for exotic particles and fields in physics. The opposite is the case.

Signal detection theory in Hilbert space

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Abstract: The Hilbert space formalism is a powerful language to express many cognitive phenomena. Here, relevant concepts from signal detection theory are recast in that language, allowing an empirically testable

extension of the quantum probability formalism to psychophysical measures, such as detectability and discriminability.

The Hilbert space formalism seems to be a suitable framework to accommodate the experimental richness of cognitive phenomena. The target article by Pothos & Busemeyer (P&B) accomplishes the impressive task of providing, in as simple a way as possible, the theoretical grounds as well as the empirical underpinnings of a probabilistic model capable of grasping many aspects of human cognition. The contribution of this commentary is to point out that important concepts arising from signal detection theory (SDT) can be easily recast into the language of quantum probability. If useful, this addition to P&B's model might be used to describe several phenomena involved in perceptual detectability and discriminability, enlarging the theoretical reach of their proposal and offering new alternatives to verify its empirical content.

SDT is a powerful tool that has been very successful in many areas of psychological research (Green & Swets 1966; Macmillan & Creelman 2005). Originally stemming from applications of statistical decision theory to engineering problems, classical SDT has been reframed over the years under many different assumptions and interpretations (Balakrishnan 1998; DeCarlo 1998; Parasuraman & Masalonis 2000; Pastore et al. 2003; Treisman 2002). However, irrespective of their formulation, signal detection theories rely on two fundamental performance measures: *sensitivity* and *bias*. Whereas sensitivity refers to the ability of an observer to detect a stimulus or discriminate between two comparable stimuli, response bias implies a decision rule or criterion, which can favor the observer's response in one direction or another.

As in standard SDT and its sequels, in this commentary sensitivity and bias are also derived from the probabilities of hits, $p(H)$, and of false alarms, $p(FA)$, defined as the probability of indicating the presence of a stimulus when it is present or absent. Two other quantities, the probabilities of misses, $p(M)$, and correct rejections, $p(CR)$, are complementary, respectively, to $p(H)$ and $p(FA)$: $p(H) + p(M) = 1 = p(FA) + p(CR)$.

To translate these concepts into the language of Hilbert space, we need to represent a perceptual state by a state vector $|\Psi\rangle$. This vector denotes the perceptual state of an observer immediately after a trial wherein either the background noise was presented in isolation ($|\Psi_n\rangle$) or the noisy background was superimposed with the target stimulus ($|\Psi_{n+s}\rangle$). These state vectors are formed by a linear combination of the $|yes\rangle$ and $|no\rangle$ vectors, an orthogonal vector basis in the present Hilbert space. The components of a state vector along the one-dimensional subspaces that represent the two possible outcomes (either the response “yes” or the response “no” to the question “was the stimulus present?”) are given by the projection of the state vector onto the subspaces for “yes” and “no,” which are spanned by the basis vectors (Fig. 1). Denoting by P_{yes} and P_{no} the projection operators (projectors) onto the subspaces spanned by the basis vectors, the components C_1 and C_2 of a state vector $|\Psi\rangle = C_1|yes\rangle + C_2|no\rangle$ are given by $C_1 = \|P_{yes}|\Psi\rangle\|$ and $C_2 = \|P_{no}|\Psi\rangle\|$.

As we can see in Figure 1, the probabilities of hits, misses, false alarms, and correct rejections are obtained by the action of the projectors P_{yes} and P_{no} on the state vectors $|\Psi_n\rangle$ and $|\Psi_{n+s}\rangle$. These probabilities can be computed by means of the following “statistical algorithm” (Hughes 1989):

$$p(H) = \langle \psi_{n+s} | P_{yes} \psi_{n+s} \rangle = \|P_{yes}|\psi_{n+s}\rangle\|^2 \quad (\text{eq. 1})$$

$$p(FA) = \langle \psi_n | P_{yes} \psi_n \rangle = \|P_{yes}|\psi_n\rangle\|^2 \quad (\text{eq. 2})$$

$$p(M) = \langle \psi_{n+s} | P_{no} \psi_{n+s} \rangle = \|P_{no}|\psi_{n+s}\rangle\|^2 \quad (\text{eq. 3})$$

$$p(CR) = \langle \psi_n | P_{no} \psi_n \rangle = \|P_{no}|\psi_n\rangle\|^2 \quad (\text{eq. 4})$$

Analogously to SDT, two measures can be extracted from the vector representation of a perceptual state: an angle, δ , which evaluates the separation between the two state vectors, $|\Psi_n\rangle$ and $|\Psi_{n+s}\rangle$, gives a measure of sensitivity; another angle, χ , which