Unstructured hexahedral non-conformal mesh generation

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Abstract

Mesh generation is a process of spatial subdivision of a generally complex domain into simple-shaped sub-volumes of pre-defined topology. These sub-volumes are usually referred to as mesh elements or cells. Elements of a mesh are connected to each other, they do not intersect with each other and cover the entire domain.

The problem of generation of computational meshes arose as a pre-requisite for numerical analysis of physical phenomena. When computational power of new computers increased sufficiently for modeling physics using discrete approaches such as finite-difference, finite-volume and finite element methods, spatial discretization emerged as a critical issue. On one hand, all requirements of numerical methods for computational meshes should be met during mesh generation. For example, these requirements may include limitations on types of elements allowed in the mesh, on mesh quality, or on connectivity between mesh elements. On the other hand, meshing tools should be capable of successful treatment of the widest possible class of geometries. These two factors provide a perpetual challenge for creators of mesh generation tools for numerical analysis.

The recent evolution of numerical analysis and design methods, combined with quickly growing range of simulated phenomena and the constantly increasing complexity of involved geometries, provide a high demand for powerful automatic meshing tools. Meshing became a necessary component of industrial design loops and a decrease in its cost is as important as that of other components. Moreover, meshing fills the gap between geometry processing in CAD systems and numerical analysis. Therefore, automation is the ultimate direction of evolution of up-to-date mesh generation tools.

Although the problem of automatic tetrahedral mesh generation has practically been solved and is successfully employed in the industry, there is still a high degree of interest in fully automatic hexahedral mesh generation. An obvious reason is that hexahedral elements are naturally suitable for numerical analysis for both finite-volume and finite-element methods. Besides, for a certain number of nodes, fewer hexahedra are required to mesh a domain as compared to tetrahedra. It means that less memory is used for the mesh and solution data storage and a shorter time is required to compute the solution. Therefore, automatic hexahedral unstructured mesh generation remains to be one of the biggest challenges among industrial demands for numerical design.
In recent years attempts to solve the problem of automated unstructured hexahedral meshing have been focused on the few principal approaches reviewed in the following sections. Among others, the so-called *Overlay* method is considered as one of the most robust automatic tools capable of dealing with geometries of practically arbitrary complexity. This approach is considered as one of the most promising directions towards full automation of hexahedral meshing process. Certain critical aspects of this approach constitute the subject of this thesis.

**Untangling concave cells.** The term untangling describes a transformation of a cell shape from concave to convex. Generally, the issue of improvement of mesh cell quality represents a combination of two operations applied to mesh cells, one of which is untangling. Its goal is to convert concave cells into convex ones via finding new appropriate positions for their vertices. Accordingly, the second operation is optimization, which is aimed at improving cell quality according to a certain quality measure. The problem of untangling consists in finding any convex cell shape, while the goal of optimization is to find a convex cell shape of best quality. The former can be solved via linear programming methods that are generally fast and not demanding.

**Optimization of mesh cells.** As the untangling process recovers only invalid (concave) cells, it is clearly insufficient for a global mesh quality improvement. Even if all mesh cells are valid, the low quality of mesh cells (for instance, too small dihedral angle or another quality measure) may degrade not only the convergence rate of computations but also the accuracy of final results. Mesh optimization procedure is aimed at direct improvement of mesh cell quality. It is based on repositioning mesh vertices to minimize a certain functional. This functional expresses the deformation energy of a cell with respect to shape of an undeformed cell. Generally, this functional represents a non-linear function of mesh vertex coordinates. Unlike the untangling procedure, solving an optimization problem is usually quite time-consuming.

**Viscous mesh generation.** Modern methods for viscous computations as well as integrated physical models often require high quality of computational meshes. This includes smoothness of viscous grids as well as control of position of the first mesh point close to wall, depending on the type of a turbulence model. Viscous layer insertion methodology developed in this thesis addresses the above issues. It combines the Laplacian smoothing with the untangling and optimization procedures in order to inflate Euler mesh cells adjacent to solid boundaries (i.e. increase their normal sizes). The inflated cells are refined tangentially until a desired number of anisotropic cell layers is inserted with their vertices being redistributed in normal direction.

**Mesh deformation.** There exists a wide class of physical phenomena modeling of which requires flow analysis in domains of shapes that vary during computation. This class encompasses multiple coupled problems with moving or deforming interfaces, fluid-structure interaction problems and many others. Solving such problems demands that a computational mesh follows deformation of domain boundaries. Therefore, two main possibilities exist. The first one is to fully re-mesh the domain every time its shape is modified. The second is to deform an existing mesh according to deformation of its
boundary. However, in many phenomena mentioned above, relative deformation of mesh boundary is small and full re-meshing appears to be too costly. Instead, deforming an already existing mesh may be quick and reliable provided that appropriate tools are available. Development of such a tool for unstructured hexahedral non-conformal meshes is addressed in the thesis. The proposed methodology preserves mesh topology and modifies only positions of vertices. Displacements of boundary points are propagated to volume mesh points. Optimization loops are incorporated in order to ensure high mesh quality.