Higher Derivative Corrections to the Abelian Born-Infeld Action using Superspace Methods

All the efforts done in theoretical physics today can grosso modo be split into two categories. The first is finding the fundamental underlying principles that govern physical phenomena and the second is trying to explain what we observe in nature starting from these principles. The quest for a theory of everything is primordially an effort find a single underlying principle. At the moment there are two complementary theories which combined give an excellent description of nature: the Standard Model and General Relativity. From this it is apparent that more work is needed towards a single underlying theory. In the ideal case we need a theory that unifies gravity with the other forces and correctly predicts the twenty-six free parameters in the Standard Model. A promising candidate is String Theory in which one assumes that elementary particles are small vibrating strings rather than point-particles. In the low energy regime, the resulting model is essentially a supersymmetric version of general relativity coupled to a supersymmetric gauge theory.

Although string theory has many pleasing features it also is spawned with various connected problems. Efforts to solve these naturally opened up a the door to the discovery of D-branes which revolutionized string theory. A Dp-brane is a p-dimensional dynamical object defined by the fact that open strings can end on it. A tantalizing aspect of D-branes is their intimate relation with gauge theories. Indeed, the effective action of a single brane in the slowly varying field limit is the Born-Infeld action which in leading order is nothing more than the action for ordinary Maxwell theory. This is the result not including derivatives on the fieldstrength of the $U(1)$ gauge field. It was shown that the term containing two derivatives vanishes and it was not until recently that the four derivative corrections where calculated.

In this thesis we study two-dimensional supersymmetric non-linear sigma-models with boundaries. We derive the most general family of boundary conditions in the non-supersymmetric case. Next we show that no further conditions arise when passing to the $N = 1$ model and we present a manifest $N = 1$ off-shell formulation. The analysis is greatly simplified compared to previous studies and there is no need to introduce non-local superspaces nor to go (partially) on-shell. Whether or not torsion is present does not modify the discussion. Subsequently, we determine under which conditions a second supersymmetry exists. As for the case without boundaries, two covariantly constant complex structures are needed. However, because of the presence of the boundary, one gets expressed in terms of
the other one and the remainder of the geometric data. Finally we recast some of our results in $N = 2$ superspace.

Leaning on these results we then calculate the beta-functions through three loops for an open string sigma-model in the presence of $U(1)$ background. Requiring them to vanish is then reinterpreted as the string equations of motion for the background. Upon integration this yields the low energy effective action. Doing the calculation in $N = 2$ boundary superspace significantly simplifies the calculation. The one loop contribution gives the effective action to all orders in $\alpha'$ in the limit of a constant fieldstrength. The result is the well known Born-Infeld action. The absence of a two loop contribution to the beta-function shows the absence of two derivative terms in the action. Finally the three loop contribution gives the four derivative terms in the effective action to all orders in $\alpha'$. Modulo a field redefinition we find complete agreement with the proposal made in the literature. By doing the calculation in $N = 2$ superspace, we get a nice geometric characterization of UV finiteness of the non-linear sigma-model: UV finiteness is guaranteed provided that the background is a deformed stable holomorphic bundle.