Characterising substructures of finite projective spaces

by

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Abstract.
In this thesis, we study several substructures in finite geometry, that is, structures contained in the Desarguesian projective space $\text{PG}(n, q)$ over the finite field $\mathbb{F}_q$.
First, we investigate pseudo-caps and weak eggs. These are the higher dimensional equivalent of caps and ovoids. We provide conditions on element induced spreads that ensure these structures are contained in a Desarguesian spread. Next, focussing on the Desarguesian spread itself, we obtain a geometric characterisation in terms of the normal elements of the spread.
Secondly, we consider linear representations $T^*_{n-1}(\mathcal{K})$, which are point-line incidence structures embedded in $\text{PG}(n, q)$ and completely defined by a point set $\mathcal{K}$ at infinity. If the set $\mathcal{K}$ contains a frame, the full automorphism group of this structure is found. Moreover, using the corresponding incidence graph, we construct new infinite families of semisymmetric graphs.
Lastly, we consider substructures in the Andrè/Bruck-Bose representation of $\text{PG}(2, q^n)$ in $\text{PG}(2n, q)$. We investigate the representation of $\mathbb{F}_{q^k}$-sublines and $\mathbb{F}_{q^k}$-subplanes of $\text{PG}(2, q^n)$ and obtain a characterisation of the ovoidal Buekenhout-Metz unital of $\text{PG}(2, q^2)$ in terms of its Baer secants.