Abstract

Group representations: idempotents in group algebras and applications to units

In this thesis, we study the group $U(RG)$ of units of $RG$, where $R$ is the ring of integers of a number field $F$. In particular, we consider the group $U(ZG)$.

First, we investigate the primitive central idempotents and the Wedderburn decomposition of group algebras $FG$, with $F$ a number field and $G$ a strongly monomial group. Next, we focus on a complete set of matrix units in the Wedderburn components of $QG$ and $FG$, with $F$ a finite field, for a class of finite strongly monomial groups $G$ containing some metacyclic groups.

We will also classify the finite groups $G$ for which, given a fixed abelian number field $F$, the Wedderburn components of $FG$ are not exceptional.

Thereafter, we study the central units $Z(U(ZG))$ for finite groups $G$. We construct generalized Bass units and show that they generate a subgroup of finite index in $Z(U(ZG))$, for finite strongly monomial groups $G$. For a specific class within the finite abelian-by-supersolvable groups $G$, we can even describe a multiplicatively independent set (based on Bass units) which generates a subgroup of finite index in $Z(U(ZG))$. For a different class of finite strongly monomial groups, containing some metacyclic groups, we construct such a set of multiplicatively independent elements starting from generalized Bass units.

Finally, we combine all results to construct a generating set of $U(ZG)$ up to finite index, for some classes of finite groups $G$. 