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A Constructivist Approach to the Problem of Induction

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Summary

The unsolved problem of induction is closely linked to "the unreasonable effectiveness of mathematics in the natural sciences" (Wigner 1960) and to the question "why the universe is algorithmically compressible" (Davies, 1990). The problem of induction is approached here by means of a constructivist version of the evolutionary epistemology (CEE) considering both, the perceived regularities we condense to the laws of nature and the mathematical structures we condense to axioms, as invariants of inborn cognitive and mental operators. A phylogenetic relationship between the mental operators generating the perceived and the mathematical regularities respectively may explain the high suitability of mathematical tools to extrapolate observed data. The extension of perceptual operators by means of experimental operators (i.e. by means of measurement devices) would lead to the completion of the classical world picture if both the cognitive and the physical operators are commutable in the sense of operator algebra (quantitative extensions). Otherwise the physical operators will have invariants which no longer can be described in classical terms, and, therefore, would require the formation of non-classical theories (qualitative extension), exceeding the classical world picture. The mathematical analogon would be the algorithmic extension of elementary mathematical thinking exceeding the axiomatic basis previously established according to Gödel's incompleteness theorem. As a consequence there will be neither a definitive set of axioms in mathematics, nor will there be a definitive theory of everything in physics.

Introduction

Thinking in terms of induction is the most elementary and the most frequently used strategy of organising our life. Be it in day-to-day life where we have to make our usual decisions on the basis of incomplete data or unconfirmed hypotheses, be it in science where we have to conceive theories on how to extrapolate empirical data, or be it in philosophy of science where we try to find a basis for teleology or determinism - inductive thinking dominates all we do, and it is the most successful of all the mental concepts men apply.

On the other hand, the obvious and uncontested success of induction is the most enigmatic problem philosophy of science was ever confronted with. Despite all philosophical efforts, we are more or less still in the same position as the one described by David Hume 250 years ago: Universal laws can be justified only by induction which he took to be unjustifiable, although natural to us. A. F. Chalmers said (1982, p. 19) "Faced with the problem of induction and related problems, inductivists have run into one difficulty after another in their attempts to construe science as a set of statements that can be established as true or probably true in the light of given evidence. Each manoeuvre in their rearguard action has taken them further away from intuitive notions about that existing enterprise referred to as science. Their technical programme has led to interesting advances within probability theory, but it has not yielded new insights into the nature of science. Their programme has degenerated". Nearly the only success achieved up to now is in clarifying and specifying the

problem itself.

The key notion in this context is what Wigner (1960) called "The unreasonable effectiveness of mathematics in the natural sciences" meaning that it is difficult to understand why so much of the complexity of the world can be described by so relatively simple mathematical formulae. So Davies (1990a) has a similar idea in mind when following an idea of Solomonoff (1964) saying "All science is really an exercise in algorithmic compression. What one means by a successful scientific theory is a procedure for compressing a lot of empirical information about the world into a relatively compact algorithm, having a substantially smaller information content. The familiar practice of employing Occam's razor to decide between competing theories is then seen as an example of choosing the most algorithmically compact encoding of the data. Using this language, we may ask: Why is the universe algorithmically compressible? and why are the relevant algorithms so simple for us to discover?" Another version of the same question is (Davies 1990b) "... how can we know anything without knowing everything?", and, more generally: "Why is the universe knowable?"

As to the criticism of the notion of induction Popper (1982, p. 4) goes even one step further: Despite all practical success of inductive thinking, according to him natural science should dispense with induction at all as it cannot be justified. His argument is that a general principle of induction can be neither analytic nor synthetic. Were it analytic it could not contribute to the growth of knowledge and therefore would not be inductive at all. Were it synthetic it would have to be justified by another inductive principle of a higher order which would lead to an endless regression.

These and many other positions concerning induction have one thing in common: They arise from our intuitive conviction that there is something existing independently which we have to recognise without having any a priori idea what it could look like. In other words: all these positions arise from the claim to organise our life by means of the category of an independent reality to be described in terms of its structure. With Popper, this is comprised in the term "growth of knowledge" to which induction has to contribute and which can be defined only in the context of some reality about which one might accumulate knowledge.

Davies is taking his stand even explicitly (1990a): "There exists a real external world which contains certain regularities. These regularities can be understood, at least in part, by a process of rational enquiry called scientific method. Science is not merely a game or charade. Its results capture, however imperfectly, some aspect of reality. Thus these regularities are *real* properties of the physical universe and not just human inventions or delusions. ... Unless one accepts that the regularities are in some sense objectively real, one might as well stop doing science."

The nearly generally agreed view that the problem of induction can and has to be solved only within the framework of an ontological reality is being given a new and surprising turn by evolutionary epistemology (EE).

The classical version (as I would like to call it) of EE (see Vollmer 1980) declares that human categories of perception and thinking such as space, time, object, reality, causality etc. result from evolution in the same way as organic elements and features do. This, in classical parlance, would mean: In the same way as organic evolution is guided by adaptive forces, cognitive evolution is said to be the result of adaptation to the independent structures of an ontological reality. Campbell (1973) speaks in terms of a "natural-selection-epistemology". The general argument goes as follows: The theories we have designed to describe the structures of reality are surely incomplete or may have other strong deficiencies - reality itself, however, has been developed as a category of human thinking just because of the

ontological character of outside reality. The fact that we think and act in terms of reality is taken as a proof that a sort of reality must exist. What is done here is to explain the formation of the category of reality by means of reference to its own content, i.e. to the existence of an ontological reality. Further to the fact that this would lead to circular inference, there is an even stronger objection: The existence of an ontological reality may, of course, have been a good reason for mental evolution to emulate it by a corresponding category of thinking. This argument, however, can not be reversed. It cannot be said that human mental phylogeny never would have brought about the category of reality if there were no such thing as an ontological reality, as long as other reasons can be found which are functionally conceivable and phylogenetically plausible though they do not refer to an ontological reality.

It will be shown that such reasons can be given. The consequence, then, is that it is no longer an epistemological must to start from an independently predefined ontological reality which is said to determine *à la longue* both the strategies of mastering nature and the theories of analysing it - a reality which was also held responsible for all kinds of teleological thinking. Abandoning the notion of an ontological reality, however, will provoke some important questions: What does it mean to assert that the category of reality is a specificum of human thinking without any ontological quality of its own? What are the evaluation criteria for theories which are said to reflect the specificity of an independent reality? Can we renounce an ontological reality without bringing the well proven methods of empirical sciences into discredit? - leading questions which deterred most scientists from tangling with the ontological point of reality. We will see that for the time being constructivist approaches seem to be the only candidates which could provide appropriate tools for coping with these problems. We will show that an appropriate version of constructivism will enable us both

a) to redeem the natural claim of EE to make reasonable hypotheses on the development of *all* mental categories - including that of reality itself, describing them as the result of an autonomous coevolution. The EE based on that kind of constructivism we will call "Constructivist Evolutionary Epistemology", CEE.

b) to reflect on inductive phenomena outside the confining and hampering context of realism.

The notion of theories

For dealing with these questions it is useful to consider theories under two different aspects: function and structure.

a) *Theories in the functional sense*: Lorenz (1971 p. 231-262) and Popper (1973 p. 164) have suggested enlarging the notion of theory towards all kinds of problem solving instruments. This would comprise physical theories in the proper sense in so far as they help us to master technical problems and to control physical nature; the inborn categories of space and time we use to interpret perceptions and to coordinate mechanical activities; limbs as instruments for locomotion; biological species as an instrument to meet the particular requirements of a special biotope; social communication and social bodies arising from it as a tool to meet the requirements of a wider social environment. All these various kinds of theories we shall call theories in the broader sense, as opposed to rationally generated theories in the usual sense such as physical theories. If this generalisation of the notion of theory is to be more than just a formal possibility, these theories must evolve according to similar principles making it possible to describe coherently biological evolution (i.e. the evolution of organic "theories") as well as cognitive, scientific, cultural and social evolution. That such a "Theorie of the Evolution of Theories" (TET) can be given - despite the long-

lived prejudice that the organic and cultural evolution differ in their very character - was shown by Diettrich (1989, p. 175).

b) A *theory in the structural sense* is considered to be a picture, an image or a mapping of an object, of an area or, more generally, of a section of nature. It is understood that the structure of the theory and the structure of the object described are partially isomorphic.

Functional theories are better the more they meet the given requirement. Structural theories, however, are better the more isomorphic they are with the structures they have to copy. It is common understanding that this is equivalent in the sense that a structural theory which is isomorphic with the structures of reality (Popper speaks in terms of truth and verisimilitude) also has functional qualities. In other words, structurally true theories are considered to be functionally helpful theories. Nevertheless and despite all practical success, that structurally true theories are usually functionally "good" ones is far from being evident and the question why they are so is far from being understood. Indeed, there is no a priori explanation why structural theories, though they are usually based on only a limited number of observations, can help us to predict correctly a mostly infinite number of new observations by means of interpolation or extrapolation, i.e. why we can reduce the boundless variety of the world to some relatively simple algorithmic structures which enable us to predict events and experiences nobody has seen or made before.

The alleged equivalence of structure and function or of truth and helpfulness is the main legitimation for all empirical science. Though we start in many practical cases from functional experiences which we try to explain a posteriori by means of structural theories, the general strategy, particularly of basic sciences, to master nature is to search for the structures of nature. This is considered as a heuristic imperative. Hence it follows that reality must be the only source of competent criteria for the evaluation of any empirical theory.

So it could be said that theories in the usual sense are teleological in character. Their progress is said to be guided by the structure of reality or, more precisely, by boundary conditions which reflect these structures, rather than that they are the result of an autonomous and independent development. Scientific evolution, therefore, must converge - not necessarily monotonously but at least asymptotically - towards a final state which would comprise a definitive and correct description of nature. Davies (1990b) sees this view as follows: "Let me express this point in a somewhat novel way. Hawking (1979) has claimed that 'the end of theoretical physics may be in sight'. He refers to the promising progress made in unification, and the possibility that a 'theory of everything' might be around the corner. Although many physicists flatly reject this, it may nevertheless be correct. As Feynman (1965) has remarked, we can't go on making discoveries in physics at the present rate for ever. Either the subject will bog down in seemingly limitless complexity and/or difficulty, or it will be completed."

What has been said here can be summarised as follows: The alleged relationship between structure and function not only means (1) that a theory's structure will determine its functional qualities, but also (2) that the structure of what we call nature will determine à la longue the theories we have to apply in order to cope functionally with this nature.

The *first allegation* is the basis of the classical notion of information: It is generally understood that the classical idea of information implies that a message is something which defines its own effect. What a piece of information will bring about is comprised entirely in itself. It is of course possible to interpret a piece of information in this way or in another, but the core-message, i.e. the meaning of it, is comprised in its structure; and the effect of a message can be changed only if this structure is changed. The classical notion of information,

therefore, is that of a structural theory comprising everything which can be deduced from the information. Particularly it would comprise what we usually call the meaning of the information. This is expressed explicitly by Hofstadter (1979 p. 165) saying (in the context of the decipherment of ancient texts) "Generally we can say: meaning is part of an object (or a text) to the extent that it acts upon intelligence in a predictable way."

As to the functional point of view, however, information has no intrinsic meaning and no inherently predefined consequence. Only the addressee will decide - mostly on grounds of special and given procedures - what he will do with a particular piece of information. That men would interpret the same signal or the same (ancient) text in almost the same way - if at all! - is simply due to the fact that they apply, if not the same language then the same metalanguage i.e. the same semantics. But it does not justify saying that the information comprised in the signal has its particular specificity which can be understood "correctly" by any sufficiently sophisticated intelligence, independently from its phylogenetic or cultural background.

Something very similar applies to the genotype in biology which is usually said to be a "blueprint" or to carry all relevant information for the construction of the organism. Actually, however, - and this is an increasingly noteworthy idea in biology (Katz 1982) - the genotype does not define or determine the phenotype in the sense that a purely mathematical analysis of genomic structures will allow us to construct the phenotype. The genome is not the blueprint we simply have to read and understand in order to see what the organism will look like. It is only the so-called epigenetic system (ES) - mainly comprised in the zygote - which performs the gene expression and which, together with the genome, will determine the structure of the phenotype. Different species usually speak a different epigenetic "language". This is why the genome and the ES of different species usually do not understand each other, i.e. interbreeding between species which are too distant is impossible. The second consequence is that the genome cannot be considered to be a picture of the genotype because the transformation from the genotype to the phenotype is made by - and therefore depends on - the ES. The third consequence is that organic reproduction has to be considered as a cyclic process which can be modified in principle at any stage: at the genomic stage where we can intervene by means of artificial mutations as well as at the epigenetic stage. So, further to classical gene technology it must be possible as well to carry out epigenetic technology. But the most important consequence follows from the double role of the ES: On the one hand the ES is the authority which performs the gene expression. On the other hand the ES itself is the outcome of genes which have been expressed by means of the ES of the parent organism. So changes in the ES may occur not only due to genomic modifications but also due to the structure of the ES's own predecessor. This relation is exactly the mathematical criterion for nonstable recursive processes. This means that long-term evolutionary processes can develop their own dynamic which does not need to depend on consecutive genomic mutations or environmental changes. These and similar phenomena could be summarized as "non-linear genetics" (Diettrich 1989, p. 165).

The *second allegation* is based on the suggestion that a problem would determine the methods of its solution, i.e. that functional adaptation would determine the structures and procedures by means of which adaptation will be achieved. This, obviously, is not the case. Horses and snakes, for example, though they may have developed in a similar physical environment, have entirely different organs of locomotion which have no structural element in common. So, the hooves of horses can not be considered, as suggested by Lorenz (1966), to be a sort of image of the steppe-land on which they live. Another example is the idea that the structure of reality would determine the regularities we perceive and which we condense

to the laws of nature. This argument raised hopes of sending some very elementary messages on typical human and terrestrial issues by means of a rocket into space. It was assumed that an universal reality would bring about at least similar ways of seeing the world, independent from what ever kind of extraterrestrial intelligence would be involved, so that, at least in principle, a rudimentary communication with beings from other planets will be possible. This view, as we will see, is indefensible.

The phylogenetic character of the laws of nature

If the category of reality is something men have developed within the framework of an autonomous evolution, this notion must be explained without reference to its own content, i.e. to the existence of an ontological reality. Otherwise we will run into circular arguments. This will require that nothing of what we perceive, observe or measure will comprise objective elements, i.e. elements which can be defined independently from any human specificity i.e. from what men are or do. Any regularity we perceive, observe or measure and which we try to condense to some general law, has rather to be identified as the result of a special physical or mental operator.

Among others, two hypotheses are possible on the relationship between the operator and the resulting regularity. Either the operator would produce the regularity concerned directly such as the genotype is said to produce the phenotype, or - and this is what we would like to propose here - the regularity is defined as the invariant of the operator in question. The latter view is well established in quantum mechanics: Features of objects have no ontological character. Their only relevant quality is that of being the invariant of a measuring operator. So, measuring apparatuses do not primarily measure the values of given variables. They first of all define what they are measuring. A length, for example, is what a so-called length-measuring device will measure. This seems to be trivial in day-to-day life - but it is not in physics. The difficulties of classical physics which led to the rise of quantum mechanics and the theory of relativity was in just having applied variables without having checked if a defining device can be constructed. In physical parlance: Physical variables or quantities need to be operationalised by means of a measuring apparatus in order to be acceptable for general theories.

To organise our life according to invariance principles is a very general concept: Geometrical figures and even physical subjects can be defined as invariants of motion. "A body is what moves together" said J. v. Uexküll (1920), and Piaget (1967 p. 152) made plausible that this invariance is the phylogenetic root of the formation of the mental category of space and spatial structures (in contrast to classical thinking regarding space and time as elementary categories and motion as a derivative one resulting from mapping time to a spatial curve).

If time figures, i.e. sequences of certain events, will occur always in the same order independently from when the first one happened, we would suppose them to be causally related. This leads to saying that causality is defined by its invariance under translation through time. Even the category of reality can be defined in this way. Reality, then, is what is invariant under all our acting and doing.

Physicists formalised this concept by means of a mathematical proof called Noether's theorem, which states that for any symmetry group which governs a certain physical phenomenon (i.e. if the Lagrangian governing the phenomenon does not change under the group transformation) there is a quantity which is conserved by its dynamics and which is defined by the generator of the group. Correspondingly, Invariance under translation in space

(e.g. physics is the same in Brussels and Vienna) implies conservation of momentum; invariance under spatial rotations implies conservation of angular momentum and invariance under a translation in time implies the conservation of energy. In other words: From the homogeneity of space follows the conservation of momentum and from the homogeneity of time follows the conservation of energy.

So, these conservation laws may well be universal in the sense that human beings will find them confirmed wherever they are in the universe. But they are not universal in the sense that they will belong necessarily to the findings of any intelligent beings independent of their phylogenetic history, for the homogeneity of time and space itself can not be defined objectively. It rather depends on the human specific mental mechanisms generating the metric of time and space. This we will analyse in more detail:

The arrow of time

Within the context of our day to day experiences we have a very clear understanding of what past and future is. Past is what embodies all the events we have experienced. Past is the source of all knowledge we have acquired. Future is the subject of our expectations. Future embodies the events which may happen and which we have to await in order to see if they really will happen. How can we express this by means of physical theories? Or, more precisely and according to the operationalisation concept: Are there devices or processes which can operationalise the terms past and future, i.e. the arrow of time?

Many efforts have been made in this direction. The result is short and disappointing (though not in the light of the CEE): In all cases where it is said that the arrow of time has been operationalised it can be shown that the direction of time was already comprised implicitly in the preconditions of the experiment. A typical example is the following: Shaking a box with black and white balls put in order according to their colour will always lead to disorder and never again to order. In physical terms: Entropy will increase in time and never decrease. Entropy, therefore seems to operationalise the arrow of time. But in this case the result will depend on what you do first, separating the balls or shaking them. First separating and then shaking will lead to disorder. First shaking and then separating will lead to order. So we already have to know what the terms before and after mean before we can do the experiment which is to tell us what before and after will mean. Another example: A hot physical body left in a cooler environment will always cool down. But this applies only if the collision processes between the atoms involved are endothermal, i.e. if the kinetic energy of the collision partners are higher before the collision than they are afterwards. If we have however exothermal processes which are characterised by the fact that the kinetic energy of the particles involved is higher after the collision, then the body will heat up rather than cool down. Here again we have to know what before and after means in order to define the collision process which will define the result of the experiment which is to define the arrow of time.

These are particular examples. I. Prigogine (1979, p. 220) has shown in a more general way that irreversible processes in thermodynamics cannot help us to operationalise the arrow of time: The existence of the so called Ljapunow-function - which is closely related to macroscopic entropy - is a prerequisite for the distinction between past and future also in microscopic systems. Unfortunately, the Ljapunow-function is ambiguous with respect to the arrow of time. It can be constructed in a way such that equilibrium will be achieved in the future as described in classical thermodynamics but it can also be constructed so that the equilibrium will be "achieved" in the past.

From all this one can make the hypothesis that in principle the arrow of time cannot be operationalised objectively, i.e. it cannot be derived from what we call nature. What past and future means, then, has to be described only by means of a sort of mental operationalisation. The following definition, for example, may be suitable: From two perceived events A and B, A is said to be before B if we can remember A when B happens but not B when A happens. Of course, past is what we can remember but we cannot remember future. This "mentalisation" of past, present and future, I think, is very close to what Einstein (published 1972) may have had in mind when he wrote to his friend Bosso "that these categories are sheer illusions".

Causality and the metric of time

Similar conclusions are suggested if we try to explain what causality is. If a certain type of event B occurs just and only if an event of type A has happened before, we infer that A is the cause of B. So we infer that lightning is the cause of thunder because there is no thunder without preceding lightning. But on the other hand, thunder is also followed by lightning, - not necessarily the same day but at some time. That the last thing we witnessed was thunder does not mean that lightning will not follow. So, from the pure topology of events we cannot say whether the lightning or the thunder is the cause. That we actually decided for the lightning has a simple reason. The time between lightning and the next thunder is usually much shorter and varies less than the time between thunder and the following lightning. But this statement we can make only if we can distinguish between shorter and longer intervals of time, i.e. if we had a time-metric, a sort of clock implemented somewhere and somehow in our cognitive apparatus. The running of the mental clock, therefore, would decide upon the causal relations we use as the constituent elements for the theories by means of which we describe what we call nature.

The laws of physics

The running of the mental clock also decides what processes we consider to be uniform in time, or, as mentioned above, it will define the homogeneity of time: If the mental clock were realized by processes which - in terms of the usual clocks - vary periodically or due to certain internal or external variables or parameters, then the moving of a force-free body for example, will not be seen to be uniform in time. Instead other processes which are physically closer to those implementing the mental clock will be regarded as being uniform, though they may be highly complex in terms of our actual human measure of time. This means that for other intelligent beings having another mental clock the classical energy will not be an invariant of their perception. I.e., the law of conservation of energy does not belong to their world. The same applies to the metric of space and the conservation of momentum. What they call their nature would be based on other invariants and therefore on other laws which are incompatible with our laws because they refer to matters we do not perceive. They would come to an entirely different description of nature which has nothing to do with how we see the world - if, at all, they would describe their experiences in terms of an independent world.

This does not only apply to time and to physical laws based on time. It applies to any regularity we perceive and therefore to any law we have derived from it. Regularities would reflect nothing but the patterns and processes predefined by our own human cognitive

apparatus. The description of what we call nature is a description in terms of our own physical and mental constitution. Nothing in our findings, therefore, can be said to be determined by an independent world. So the concept itself of an independent world is - from the epistemological point of view - of no value. It is a scientifically redundant notion.

A similar conclusion is true even for the laws of logic. They are not universal in the sense that they are valid in any possible world as Leibniz said. Rather they depend on mental definitions, i.e. they are the outcome of certain internal structures and processes predefined in our mental apparatus. As they are equal on phylogenetic grounds for all human beings - otherwise we could not logically argue with each other - we are led to believe erroneously that they are equal not only for humans but necessarily also for any sufficiently intelligent being.

Hence we can conclude: there is neither a *physical reality* in the sense of something which can be described in terms of independent and objective laws of nature; nor is there what one may call a *notional reality* in the sense of synthetic a-prioris and of objective laws of logic, i.e. of statements which will be necessarily accepted by any sufficiently intelligent thinking being.

To return to a widespread objection: The fact that all the regularities and laws of nature identified by men are home made in the sense explained here (the antropic explanation), does not conflict either with empirical sciences when using these laws or with how we would tackle daily life affairs. Not only are the roots of our problem solving knowledge home made but also the problems themselves. There are no objective problems which would require objective theories. Problems are nothing but special problems of special individuals in special situations. All human striving to come to a better life can be reduced to achieving certain perceptions and avoiding others. This is what our special problems are and this is why theories can succeed in providing us with "better" perceptions only if they deal with the relations between home made perceptions (here, of course, they do remarkably well!). Objective theories, however, based on the structure of an external reality (what ever that would mean) are empirically unverifiable. So they are entirely irrelevant and useless - and so are all efforts to find them. In other words: Science must be antropic. Otherwise it would deal with matters men have no relation with.

The mental apparatus decides upon the specificity of all kinds of things we perceive. That we would describe our environment primarily in terms of visually-perceived concrete physical objects is a direct consequence of the phylogenetic decision to deal just with the invariants of moving. "An object is what moves together" as Uexküll said. This is by no means a physical or a psychological must. It would also have been possible to base the organisation of our lives mainly on the invariants of other operators which may not even commute as we do in quantum mechanics. So, whatever we speak about as the subject of our perception is predefined in our mental apparatus, mainly in terms of invariants of certain processes.

This also applies, as we have seen, for the regularities we perceive in the context of objects, and which we condense to physical theories. A special theory is that of (classical) kinetics. This theory states that there are physical mass points having an identity which is defined by the continuity of their motion in a 3-dimensional metric space (metric means that a distance is defined for any two points). I.e., Identity, as something lasting in time, is defined as the invariant of motion. Kinetics, though this would be a conceivable result of the mental evolution as well, does not speak in terms of different mass points coming into existence for just one moment, one after another in a spatial order according to what we call motion (such as the seemingly moving light spot on a TV screen). In such a theory the mass points would

have no identity lasting in time, nor is explained what motion of a mass point would mean. What our mental apparatus within the context of moving actually does is to attribute different positions of the same body to the scale of time rather than to attribute different bodies to different positions. Metrics, motion and identity, therefore are notions conditioning each other.

The phylogenetic decision towards a kinetics based on the categories of motion and identity is probably the main reason for the major difference of what we call space and time. Time is said to flow in an irreversible way; Nobody can recall a thing of the past. We cannot move between two points in time in either direction. But we can do so quite well between two points in space. If we say we travel from point A to B and then back to A, we mean that the A where we started before coming to B, and the A at which we arrived after leaving B, are not only equal but identical. To say so, however, is possible only if we can distinguish between "equal" and "identical" and if what we call identical is not influenced by our travel, i.e. if identity were defined as the invariant of motion. But exactly this is the case. Only on grounds of that definition can we call a change in spatial positions to be reversible, or more precisely: Only on grounds of that definition can we distinguish between the repeated return to the same A and a travel along a sequence of equal As, i.e. between periodicity in time and space.

Another statement of phylogenetically evolved kinetics is that time is one-dimensional and space three-dimensional. The usual argument is that our perceptual space is mainly a visual one which has three dimensions due to the three degrees of freedom our eyes have (stereoscopic fixation included). The perception of time, however, is mainly acoustically oriented and hence one-dimensional as our ears can perceive only sequential data. This view we have to refute for two reasons:

1. Acoustical data too have many degrees of freedom (tone pitch, volume, tone colour, harmonic composition or any other integral measure). A priori there is no reason why we do not use these variables to form a pluri-dimensional "sound-space". The data we perceive visually have even more degrees of freedom. To select just four of them to perform a space-time continuum which enables us to identify separate objects and to attribute the rest of the data as properties to these, is by no means a logical must. An alternative would be to integrate, say, n objects into a single object moving in a $6n$ -dimensional phase space (this is what physicists often do in statistical mechanics). Even the opposite would be possible, to "perceive" three objects moving in an accordingly coordinated manner in a one-dimensional space each, instead of one object moving in a three-dimensional space. This is not as exotic as it may sound. In stereoscopic vision our perceptual apparatus does something very similar. Usually, the retina of the two eyes is exposed to different light stimuli. A more "naive" brain would infer from this that the two eyes are confronted with two different objects. The strong correlation in shape and action of these "objects" such a brain would regard as an explicit "law of nature". Our actual visual data processing, however, would compress the two perceptions into just one. The difference between the two pictures is no longer directly perceived but transformed into a new property of the subject in question called distance. One may argue that this can hardly be the outcome of an autonomous phylogenetic decision as the obviously real three-dimensional character of our environment seems to be well confirmed by tactile perceptions. Actually, however, this coincidence does only say that the visual and the tactile senses "decided" to interpret their respective perceptions by means of the same theory which would enable them to substitute each other when necessary (In the dark e.g. we can explore the shape of a body also by touch). Similarly physicists and chemists agreed to interpret the phenomena of their fields within the framework of the same atomic theory. Therefore, chemical reaction kinetics can be analysed by means of the thermodynamic

methods of physics. Summing up, it must be said that neither the constitution of our eyes nor the construction of our hands have anything to do with the fact that our perceptual space has just three dimensions. What we "see" are primarily two-dimensional pictures which are even not invariants of motion. That we divide these impressions into invisible objects being invariant under motion on the one hand, and into their visible but covariant pictures on the other hand, and that we consider the visible changes of these pictures to be caused by relative motions within a space which, just for that purpose, has been given a third dimension - all this is purely a "software" matter of our cognitive apparatus which has nothing to do with any "hardware" of our eyes.

2. To speak in terms of the motional degrees of freedom of our eyes and hands as a reason for the three-dimensional character of our perceptual space, means using implicitly the notion of space in order to explain just this notion, i.e. means to run into circular arguments.

The notion of reality

Despite all arguments from the CEE to declare reality to be a scientifically redundant notion, there is no doubt that reality is one of the most elementary categories of our thinking, and that not to ignore the facts of reality is one of the first things we have to learn in our life. So, the question will arise: Why did the category of reality achieve so much importance?

The function of reality can be seen under two aspects. The first one is the *phylogenetic aspect*: What might have been the "reason" why evolution has brought about the mental category of reality? The second one can be called the *ontogenetic aspect*: What is the role reality plays with individuals using this category when developing strategies to organise their lives?

Organic evolution, as a nearly general rule, proceeds by modifications of the highest existing hierarchical levels. The more elementary levels remain largely unchanged. Metazoal evolution e.g. would mean altering arrangements of cells leaving the cells themselves mainly unmodified. The reason is simple. Organisms are highly specified networks of special interactions between organs and - inside organs - between cells. These networks constitute very close boundary conditions for the further development of the more elementary structures. Accidental modifications of the elements would nearly always destroy all the functioning established before. Nature found (implicit) ways within the reproduction mechanisms to keep genetic mutations from intervening in these functional essentials ("genetic fixation", see Riedl, 1975).

The same must hold for the formation of higher theories based on a hierarchy of more elementary ones. The most elementary theories (above the level of conscious recording) are observations, i.e. the interpretations of perceptions. These interpretations we are accustomed to use in order to construct higher theories have to be protected from any further individual modification or improvement we may envisage. Otherwise all the daily experiences we have acquired and accumulated become useless and all empirical sciences have to be reconstructed again and again from scratch. The notion of learning is meaningful only if the elements on which learning is based can be considered to remain unchanged. In order to achieve this, phylogeny has developed a brilliant trick: The theories to be protected are put in a special "box". This box we are told is "external" i.e. it is by definition outside the range of our individual efforts. We call this box reality and we are convinced that we have no access to the box and no means to modify its content. Of course, this is exactly what we understand reality would mean: something we cannot change and we should not ignore in pursuit of our aims, i.e. the structures of reality are considered to be sacrosanct (Diettrich 1989). So, from the

functional point of view, the mental category of reality acts as an instrument used to protect the established interpretations of perceptions, i.e. to immunize observations. This is why observations qualify to constitute a special class of higher theories we call empirical.

For the realist also reality plays a functional role, though of an entirely different character. In his eyes reality provides the evaluation criteria for any empirical theory. Though these criteria are not given explicitly and are said to act rather by means of occasional falsification, the realist is convinced that reality will guide the scientific development *à la longue* towards a set of theories or a "theory of everything" which are no longer subject to falsification.

If we deprive the realist of reality we have to explain to him what the boundary conditions for the formation of theories are and where else they come from. What scientists (realists included) from the functional point of view are interested in are theories which allow a maximum of correct extrapolation, i.e. theories of optimal forecast-power. That the realist believes that theories can forecast observational data only if they correspond to the structures of reality is a consequence of his position which would become irrelevant if we can provide also other reasons why theories allow extrapolations.

There is still another aspect under which we can see the functional role of reality. In German there are two terms which nearly mean the same: *Wirklichkeit* and *Realität* (actuality and reality). *Wirklichkeit* comes from 'wirken' (to effect). These two notions can be used in the following sense: *Wirklichkeit* refers to the experience that all our action is subject to boundary conditions we cannot influence, that our perceptions have specificities we cannot change, and that the course of our life will comprise elements we can neither control nor predict. But *Wirklichkeit* does not say why it is so and where it comes from. This is the object of a special theory we acquired phylogenetically and which we call reality. This theory of reality says that the subject of our consciousness is split into a so-called internal and an external world and that similar splitting provides the external world with a certain structure, and that this structure is responsible for the specificity of *Wirklichkeit*. Reality, so to say, is a special theory of *Wirklichkeit*. Another one we will present here.

The evolution of our world picture

As to our relation with the external world we use two basic categories: 1. **Perceptions** which would tell us what the world looks like. We use perceptions to form theories which we expect to tell us what we have to do in order to achieve certain goals. 2. **Actions** which would alter the world and therefore would lead to altered perceptions. The distinction of these categories, however, is not without ambiguity. What we call a perception, as we have seen, is defined as the result of the application of a mental operator, i.e. of an action. On the other hand, what an action is and does can be defined only by means of the perceptions it will produce. An unambiguous and independent distinction of perception and action is possible only within the context of an external world by means of a directed interrelation of the subject towards the world (= action) or the world towards the subject (= perception).

Properties of objects, as we have seen, are defined by actions or operations. These can be mental operations generating the "directly perceivable" features of our environment, or the measurement processes of physics. On the other hand, there are cases where we act upon objects not in order to define properties or to measure a value but just in order to change the properties or values in question. We therefore have to distinguish between properties which are invariant under "usual" operations and which, therefore, we can use to characterise objects, and those which would be altered by these operations. This procedure is not

unambiguous. It depends on which operations we use to constitute and therefore to define objects (we will call them *defining or measuring operators*) and which we use to modify the objects defined previously (we will call them *modifying operators*). These two operators must not necessarily be different. Let us take a hammer. It is an operator primarily designed to alter certain objects. But also a hammer in its quality as an operator has invariants: objects and properties which would resist the hammer's strokes of a given strength. The hammer, then, can be used to measure mechanical properties such as the strength of a material. So, both perceiving and acting mean to apply operators. The essential difference is that in the case of perception we ask for the invariants of the operator in question, i.e. what remains unchanged under the application of the operators, whereas in the case of action we ask for what changes under the operator's influence. Our inborn world picture, therefore, depends on the phylogenetic decision on which operators and which invariants we will use to construct the cognitive reference frame we apply for the world's description.

An example: it is a biologically fixed cognitive convention that we use visual observation together with our moving as a defining operation defining three-dimensional objects. It would also be possible (though not phylogenetically realised) to consider moving as a modifying operation "acting" explicitly on the objects and "distorting" them in reality according to what we call perspective phenomena. Such a concept would have had the advantage that objects are what they are seen as and that they do not have to be derived in a very complex manner from a variety of different perspective pictures. A disadvantage would be that the visually perceived properties of objects would vary with position and, therefore, with the observer, i.e. objects of that kind would not be galilei-invariant. As interpersonality is the main requirement for the constitution of the category of reality (what is different for different observers can hardly be objective), physical objects can contribute to the description of an independent world only when portrayed in terms of galilei-invariants. This can be considered as the very reason why our cognitive apparatus provides us with the ability to identify galilei-invariants. It has not, however, endowed us with the ability to identify lorentz-invariants as the velocities for which this would be relevant never occurred in human phylogeny. A more recent (scientific) decision of that kind was in the theory of general relativity to replace the effect of gravity changing the motion of mass objects by the effect of the curvature of space, i.e. to speak in terms of a kind of four-dimensional perspective phenomena rather than in terms of explicit interacting forces.

One of the earlier notional splittings in cognitive evolution is that into *observational* and *theoretical* terms. Perceptions can be considered as kinds of unconsciously performed theories by means of which we structuralise the sensational input - in the same sense as we apply consciously constructed physical theories in order to structure the relations of perceptions. The so-called observational terms which describe what we see directly, and the so-called theoretical terms which comprise special interpretations, are both theories in so far as they are "man-made". The only difference is that observational terms have developed phylogenetically in the unconscious parts of the human brain, whereas theoretical terms are the outcome of conscious and rational efforts. Then, the old dichotomy of observational and theoretical terms is reduced to a rather secondary difference. Nevertheless observational terms remain privileged as the basic elements of any higher theories in so far as we can modify theories according to observational data, but we can not modify the genetically fixed mental operators and their invariants according to the requirements of special situations. (For future high-speed astronauts e.g. it might be useful to have the inborn ability to identify lorentz-invariants, and a sound engineer might be interested in the ability to perceive spectral acoustic data explicitly and not only in the form of tone colour.)

If we base empirical theories on observations, as we actually do, and if observations are theories as well, then the evolution of science is an entirely internal matter between theories. What ever we call the structures of reality, it must be comprised in the more elementary theories upon which we found higher theories. Reality, so to say, is the outcome of its own history. This will allow us to see the realist's main argument in another light: The basic experience of all men is that our perception contains regularities we cannot influence. So, they must be objective, the realist infers, and hence it is legitimate to try to condense them to the laws of an objective world. We, here, concede that we have indeed no means to influence the regularities perceived nor can we alter what we call the laws of nature - but only so far as the present is concerned. In the past, as we have seen, we contributed well to the shape and form of the regularities we identify insofar as the generating mental operators are the outcome of an independent evolution (or co-evolution). For example our phylogenetic ancestors explicitly designed the law of energy conservation when they decided upon which type of physical mechanism should emulate the mental clock defining the metric of time. So, the regularities men perceive and condense to general laws represent nothing but their own previous history.

Induction and the compressibility of observational and theoretical terms

Perceptions (and observations) are related to each other according to what we call the regularities perceived. These regularities, as we have seen, are the outcome of special mental operators. A (scientific) theory on the relation between observations, therefore, can be "true" (i.e. it can extrapolate the data observed correctly) only if it would emulate the generating mechanisms. But how can we emulate these mechanisms if we do not have any access to the brain where they are implemented and if we have no means to analyse them otherwise? What we have is nothing but mathematical methods which - astonishing enough as Wigner said - would work very effectively in helping us to extrapolate observational data. Then, the conclusion is near at hand that there is a certain homology between the mechanisms generating mathematical, logical and other theoretical terms and those generating observational ones. This would explain, of course, why observational extrapolation (i.e. waiting for the observations expected or doing the experiments required) may lead to the same result as the mathematical extrapolation of observed data does. A helpful contribution for the solution of the problem of induction, therefore, were plausible hypotheses on a common metatheory of mathematics and observational terms.

The stated equivalence of observational and theoretical terms requires that we approach mathematics and logic under the same constructivist aspect as we do with the empirical world. There is already a certain tradition of constructivist approaches (see Lorenzen, 1975) having in mind mainly a better foundation of mathematics: Only if we knew how things came up could we understand why they are as they are. Unfortunately it is not enough to find a "generative mathematics" which generates all the mathematical rules or regularities we know as there is no guarantee that it would also generate those we may still find in the future. The only guarantee for generally succeeding is that we find a solution which emulates the actually implemented mental mechanisms. This generative mathematics, however, as well as Chomsky's generative grammar, is inaccessibly sited in the subconscious parts of cognition. All we know and all we have access to are their results. From them, unfortunately and as a matter of principle, we can not conclude the generating mechanisms. This is why it is so difficult to concretize generative grammar producing more than just one or two grammatical regularities or rules.

To deal with the compressibility of mathematical terms means to pose the question: Why can we describe the results of rather complex mathematical operations by relative simple expressions? How can we extrapolate ordered sequences of mathematical operations by explicit formulae, i.e., why does the principle of mathematical induction work? That this is a serious problem is known - at least in principle. Though mathematicians generally acknowledge that Peano by means of his five axioms has considerably contributed to understanding the world of natural numbers - particularly the fifth "If the natural number 0 has some property P, and if further whenever n has P then so does $n + 1$, then all natural numbers have P" is the basis of mathematical induction, one of the most important procedures in practical algebra; but Hofstaedter has rightly remarked that this does not provide a criterion to distinguish true from false statements on natural numbers. He asked (1979, p. 229): "..., how do we know that this mental model we have of some abstract entities called 'natural numbers' is actually a coherent construct? Perhaps our own thought processes, those informal processes which we have tried to capture in the formal rules of the system, are themselves inconsistent!" Well, at least in the constructivist context, they are not inconsistent as this term is not explained there. But the possibility remains that the formal rules we have established do not correctly or completely emulate the informal thought processes (i.e. what we called mental operators). The ongoing success of mathematical sciences, however, make it rather probable that mathematics is a fairly good theory of what the mental operators can bring about. It may even be a correct or true theory if the mental operators in the course of cognitive evolution contributed implicitly to their own conscious formalisation, i.e. to the development of mathematical and logical thinking. In other words: Mathematics succeeds by means of compressing theoretical terms (e.g. by means of mathematical induction) because the mechanisms of generating theoretical terms and those compressing them are closely related to each other due to a special cognitive co-evolution having the effect that compressed and uncompressed terms behave alike and therefore are interchangeable.

The fact that a large number of empirical data can be described by a relatively simple mathematical formula, by a simple picture or regularity or by just a few words (i.e. by a theory in general), we explained by their compressibility. On the other hand we can consider these formula etc. to generate the data in question in the sense that we can derive them from the generating theory. Within the framework of constructivism, however, there is nothing that is not generated, either by a physical or biological process, by a theory in the proper sense or by a mental operator generating what we perceive as regularities or laws. Compressibility, therefore, is not a special feature of some data or entities we have to investigate or to wonder about. It is rather the central characteristic of constructivism. The generating mechanisms (and only they) can tell us how we have to extrapolate given data or what we can conclude from certain observations, i.e. how we can apply mathematical or empirical induction. Without generating mechanisms neither extrapolation nor induction is anything but arbitrary and therefore useless and meaningless. From the fact that we may have seen up to now only white swans nothing can be concluded, particularly nothing on the existence of black swans - except we have a theory or hypothesis telling us why for example other colours are not possible or do not exist.

The difficulty of classical approaches towards the problem of induction follows from the idea that the operators generating the regularities of our perceptions are exclusively non-mental external mechanisms. According to this it is generally understood that we have to extrapolate data from celestial mechanics according to the effect of gravitational forces as contained in Newton's laws. But we find it strange to understand why we usually succeed in extrapolating a number of sensual data perceived according to a regularity identified by

means of nothing but the data given themselves - as if the regularity of the past data and of those to come were caused by the same reason. But exactly this is the case. There is of course a causal reason generating these regularities, but it is not an external one as gravitation is said to be. It is rather the internal mental operators generating the regularities in question. This is the very legitimation for empirical induction. As this applies for any kind of regularity, so also the laws of classical mechanics as described by Newton are nothing but the emulation of mental operators by means of what we call explicit external forces.

Physical extensions of perceptions

The mental relationship between perception and mathematical thinking is underlined by a particular (and rather unexpected) view linking together the existence of non-classical physical theories and Gödel's incompleteness theorem:

Typical for most empirical sciences is the use of instruments and measurement devices by means of which we extend the range of natural perception in a similar way as we extend our inborn physical capabilities by means of tools and engines. Here we have to distinguish between two important cases (Dietrich 1994a)

We will speak of **quantitative extensions** if the perception operators and the measurement operators are commutable in the sense of operator algebra. In this case both operators have the same spectrum of invariants. This means that the results of the measurement operations can be presented in the terms of invariants of the inborn cognitive operators, i.e. in terms of our classical world picture.

We will speak of **qualitative extensions** if perception operators and measurement operators are not commutable. Then, the results can no longer be presented in a classical manner and require new, non-classical theories which accept classical theories only as approximative approaches in certain borderline cases. As the set of possible measurement devices is in principle unlimited, it can be never excluded that qualitative extensions of previously established operators will bring about modifications of the previously established world picture and of the theories comprised. So there will be no definitive world picture and no definitive 'theory of everything'. There will be no objective laws of nature and those laws we have 'constructed' in a human specific way will converge towards a definitive set of laws only within the context of a limited set of operators, i.e. if we resign from further unlimited experimental research. What we actually do if we do science is that we construct a world of which we believe that we analyse it by doing science. Analytical in the sense of deepening our knowledge, however, is science only within quantitative extensions. If qualitative experimental extensions provide us with new theories and if new theories will bring about new questions to be replied experimentally, than what we call the progress of science we can consider as a principally endless co-evolution of theories and experiments. A typical example is the increase of the elementary particles' zoo with the increasing energy applied. The evolution of theories and knowledge is generally not predictable since new results would not determine their theoretical interpretation, nor would open theoretical questions determine the experimental measures to answer them. New developmental lines have been created very often in the history of physical theories. Fresnel's interpretation of refraction phenomena of light by means of a wave theory (1816) led to the idea of the world ether and to the Michelson experiment, from there to the theory of general relativity, to the mass-energy equivalence and from there eventually directly to modern elementary particle physics. Fresnel's decision, however, was not a logical must. Quantum mechanics has shown that neither the corpuscular nor the wave aspect of light have an ontological quality. They are

rather purely theoretical concepts - an idea which in principle could have been derived already from the work of W. R. Hamilton (1805-65). Nobody could say where we would be today if Fresnel and his time would not have embarked on the wave theory. May be we would have neither particle physics nor nuclear energy.

Let us come back to the question of Davies, Hawking and Feynman as to whether a theory of everything is possible. According to what has been explained here we may be tempted to give an affirmative answer. If all regularities and natural laws are defined as invariants of mental operators an in-depth analysis of the brain's hard- and soft-ware seems to be the very clue for any physical law men could find. First of all, however, this would provide us only with those laws based on simple observations of the unaided sense organs such as the laws of classical mechanics. It will of course not mean that the results of elementary particle physics are encoded in the brain's structure - (though I have some hopes that further reflections on the CEE will allow us to understand better the "nature" of the constants of nature). Experimental facilities which can tell us matters the unaided sense organs would not see, can be regarded as artificial extensions of sense organs with additional or different invariants which would depend on the technical structure of the facility and which obviously can not be derived from brain analysis (Dietrich 1990). Such an experimental extension would cause new theories comprising the new invariants. From there new questions would arise causing the production of new experimental facilities which may have new invariants, and so on ad infinitum.

This position differs from realism where theories *à la longue* are determined entirely by the structure of reality as well as from that of nearly complete arbitrariness which Davies (1990b) believes to follow from the antropic explanation: "(Why bother with science at all when an antropic explanation for almost any feature of the world can be cooked up?)" This is not a necessary conclusion. The antropic explanation does not deprive the formation of theories of all restrictions. Theories, when allowing predictions - and this is what they first of all must do - do not have to mirror the structures of an independent reality. Rather, they have to emulate the generating mechanisms for perceptions, both the natural ("antropic") ones acquired phylogenetically and the artificial ones called experimental facilities.

The notion of a theory of everything is equivalent to the notion of reality characterized by objective laws of nature. Further to all what has been said here, such a notion cannot be defined operationally. To require of reality to have structures which are independent from all human action, i.e. structures which are invariant under all possible operators, would deprive reality just of the specificity which is needed to be a non-trivial notation and which can result only from being invariant under particular operations. In other words: the operator that is to operationalise the notion of reality has to be commutable with any possible operator. Unfortunately, only the trivial unity operator does so. So, what we can operationalise at best is, for example, a kind of current reality which refers to all the operations applied up to now (rather than to all possible operations), i.e. a reality which reflects all the perceptions and experiences man has ever made. This is just what we do when we speak in ontological terms about a reality which - according to our current knowledge - has this or that structure. By this it is evident that what we call reality cannot be brought about by adaptation to an independently existing or ontological reality. It rather suggests that the very purpose of the concept of reality is to immunise as far as possible acquired and proven knowledge as a standardised basis for further development. Cognitive evolution realised this by "telling" us that all knowledge has to be understood as knowledge about reality and that to ignore or to try to change the so called facts of reality would be "unrealistic", the most pejorative attribute for any scientific or common effort. Reality,

therefore, is the "cognitive burden" we have to acknowledge with when to further develop our life competence, similar to the so called genetic burden (Riedl 1975) organic evolution has to recognise. In either case the established structures define the boundary condition for their future advancement.

The algorithmic extensions of mathematical thinking

As already mentioned, CEE requires that not only observational terms have to be defined operationally but also mathematical and logical terms. If the cognitive operators generating our perception are phylogenetically related to those generating mathematical thinking, then their respective results, i.e. perceived and mathematical structures must show certain similarities. This would explain why we can simulate the extrapolation of perceptions (i.e. predictions) by means of mathematical extrapolations and why the mathematics we use is so well suitable for the description of what we call nature. In this context Davies (1990) speaks of the 'algorithmical compressibility of the world' and Wigner (1960) of the 'the unreasonable effectiveness of mathematics in the natural sciences'. From the classical point of view (i.e. within the theory of reality), however, the algorithmic compressibility of the world cannot be explained, and neither, on the same basis, can the success of induction.

If there is really a relationship between mathematics and perception as postulated here, then the phenomenon of qualitative extensions must occur also in mathematics (Diettrich 1994b). This sounds strange, but there is some plausibility behind this idea. Similar to the operators generating sensual perception which can be extended by physical facilities, the mental operators generating our elementary mathematical conceptions can also be extended through higher and more complex mathematical calculi. This is what mathematics does as science. Insofar as the higher mathematics used is based on appropriate axioms, i.e. (in CEE parlance) on axioms which emulate correctly the cognitive operators concerned, there is no reason from the classical point of view to believe that this will lead to "non-classical" statements, i.e. to statements which can no longer be formulated within the syntax constituted by the axioms concerned. This view substantiated the confidence in Hilbert's program of the complete axiomatization of mathematics - or, in the terms used here, the confidence that mathematics can extend itself only quantitatively.

From Gödel, however, we know (see the summary of E. Nagel¹⁷) that there are mathematical procedures which, though entirely constructed by means of well proven classical methods, will lead to statements representing a truthfulness which can no longer be derived from the axioms concerned. Mathematics (of the kind we know) has turned out to be as incomplete as classical physics. In either case, nothing but the application of well-trying and sound methods and procedures can lead to results which cannot be extracted from the foundations of these methods and procedures. We must therefore conclude that we cannot be sure that there will be no surprises of a similar kind in the future. Indeed: just as experimental operators, though constructed entirely according to the rules of classical physics, may lead to results which cannot be described in classical terms, there are also mathematical calculi which, as shown by Gödel, though based entirely on well tested axioms, can lead to statements which cannot be proven within the context of these axioms - and this can happen again and again. So we have qualitative extensions in physics as well as in mathematics and we can define accordingly:

We will speak of **quantitative extensions** if the truth value of the terms achieved can be derived from the axioms used.

We will speak of **qualitative extensions** if the truth value of the terms achieved

cannot be derived from the axioms concerned though the calculi used are completely based on these axioms. In this case the axioms themselves have to be extended in order to make the truth value in question derivable.

Qualitative extensions, be it in physics or mathematics, are purely emergent phenomena which cannot be predicted as they, by definition, cannot be derived from previous knowledge. The blueprints of quantum mechanical devices are entirely classical in character and nothing provoked the idea that the results they may bring about could no longer be interpreted within classical theories. The same applies to mathematics. There is no general criterion telling us whether a given calculus will exceed its own axiomatic basis.

With this, the existence of non-classical theories in physics and the incompleteness theorem of Gödel are homologous cognitive phenomena. Neither is there a definitive set of physical theories (i.e. a theory of everything) explaining and describing all (also future) physical problems nor is there a definitive set of mathematical axioms determining the truth value of all possible mathematical statements.

As to qualitative extensions, the only difference between the physical and the mathematical situation is that we already have in physics two non-classical theories (quantum mechanics and special relativity) and that we can say precisely under what conditions we have to apply them, namely (simply spoken) in subatomic areas and with very high speeds. In mathematics we only know from Gödel's theory that there must be non-classical phenomena, but we do not know what they are and, more particularly, we cannot say which operations would bring us out of the classical domain. Is it the notion of cardinal or ordinal numbers, or the notion of set or of infinity, or is it the combined application of these notions which constitute the cause of non-classical mathematical phenomena? Will logic turn out to be as incomplete as physics or mathematics? And what will happen as we deal with more and more powerful computers? Up to now we do not know. But when we will, we will have modern, non-classical mathematics as well as physics.

The astonishment of mathematicians with respect to Gödel's proof continues, unbroken. Literature is full of respective manifestations. Among others the explanation was proposed that the brain's action cannot be entirely algorithmic (Lucas 1961, Penrose 1989). Further to the fact that it is not quite clear what in a neural network such as the brain could be non-algorithmic, this kind of reasoning is not necessary at all. What follows from Gödel's proof is only that what certain mathematical calculi can bring about is not necessarily the same as what a combination of them could generate. It is as if physicists believed that physics cannot be entirely natural because apparatus constructed according to the laws of classical physics would not necessarily reproduce the laws of classical physics as seen in quantum mechanics.

In contrast to physicists who suggested as an explanation for their respective experiences that they had happened to come into domains of nature where other and unpredictable laws would rule, mathematicians hesitated to admit the idea that mathematical research is empirical in the sense that it would lead to really new discoveries which by no way could have been expected, even not a posteriori. If mathematics had its own specificity at all as included in the notion of Plato's reality, then, according to general mathematical understanding, this must be something which is included in the very rudiments and which, from there, would determine all possible consequences. In other words: if there is such a thing like Plato's reality it must reveal itself by the fact that a consistent mathematics can be based only on particular axioms (the analogy to the laws of physical reality, so to speak). Once they have been found - as per Hilbert's conviction - they would settle once and for ever the "phenotype" of all future mathematics. Mathematics, then, would be nothing but a kind of

craft filling up the possibilities defined by the axioms identified - similar to physics which, according to prevailing understanding could do nothing but look for the applications of the "theory of everything" once it has been found.

In the beginning it was hoped, that extending or modifying the axioms in view of the unprovable statements concerned could solve the problem. Unfortunately the new axioms would be in no better situation, as for any set of axioms unprovable statements may be found. This applies also for physics. Of course, we can modify theories according to 'unprovable' phenomena, i.e. new phenomena which cannot be formulated within the existing theories, and we did so when establishing quantum mechanics - but this will provide no guarantee that similar things will not happen again and again. So, neither in physics nor in mathematics can a 'tool for everything' be found by means of which all problems concerned, present and future, can be solved in a purely technical or formalistic manner.

The relationship between physics and mathematics as suggested by CEE constitutes a certain heuristic balance. Experimental physics is no longer privileged in providing information from the outside world, whereas mathematics has to set it in theories. Instead, hopes are reasonable that a possibly successful study of non-classical mathematical phenomena could be a key to better understanding non-classical phenomena in physics too - and vice versa. In a way, physics and mathematics can see each other as very general theories. So, mathematics could outgrow the role of an auxiliary science, which it has hold from the outset of empirical science, into the role of an heuristic partner of equal rights. Strictly speaking, this has already happened. Of course, that we consider the world to be algorithmically compressible reflects nothing but the suitability of mathematics for prognostic purposes in physics. This is what physicists call "the unreasonable effectiveness of mathematics in the natural sciences" which, in the light of CEE, might well be reasonable.

The homology of moving and counting

In order to verify the homology of the mental roots of empirical and mathematical thinking which we said to be the reason why observational terms can be emulated so well by theoretical terms (as manifested in the problem of induction) we must not refer to qualitative extensions. It can be shown that the relatively elementary categories of moving and counting are homologue in physics and mathematics.

Let us, in analogy to Peano, generate what we call integer (ordinal) numbers denominating a certain position within a series of equal elements, roughly speaking by defining that each integer number n has just one successor n' and just one predecessor n . Adding a given integer number p to n means that we have to perform the successor p times. Then, p is a cardinal number as it indicates the amount of processes to be applied rather than a topological position. Let us call $+p$ (or $-p$) the operator performing p times the successor (or predecessor) of n and by this generating the ordinal number m . Each ordinal number, then, can be understood to be generated from another one by means of a counting operator. Even when performing the direct successor as to be done in ordinary counting we have to indicate that we do this just once (rather than several or no times). Also here, "one" is first of all a cardinal number. (counting operators, so to say, tell us how far to proceed at the scale of numbers.) But where do we have to start when this is to be a procedure defining ordinal numbers? There must be a first ordinal number to which counting operators can be applied. Peano bypassed this problem by stating axiomatically that 0 is an ordinal number. From the constructivist point of view, this is hardly a solution. What we need are plausible assumptions on a mental generator which could help us to avoid axiomatic settings of ordinal numbers.

Concrete counting first of all means identifying the subjects to be counted. Moving, as we have seen, was a way of defining the identity of subjects. But how can we define or identify countable subjects if there is no movement? Here we need other operators generating countable subjects in the form of their invariants. In general, what we will declare to be a subject will depend on the operator applied. Many of the existing operators have been established phylogenetically in such a way as to have the same invariants as the moving operator has, i.e. once something is identified as an invariant of moving, it will also be identified when at rest. Generally, however, and particularly in new cognitive territories, what we may identify as a subject it is entirely open. It will be a matter of what kind of cognitive operators we apply.

This can be explained as follows: Let us start with what a certain operator has generated. This we have to consider as the domain of definition D for any further consideration or acting. D has no structure but that generated by an operator, say Z , applied to D . The result may be U_a . Z applied to U_a may lead to U_b and so on. So we will get a series U_i with $i = a, b, c, \dots$; Let us call Z a counting operator. It would generate the subjects to be counted as well as the order in which they will appear: Each U_i has just one successor $U_{i'}$ with $U_{i'} = Z(U_i)$. U_a is the successor of D . If there is a j with $U_{j'} = U_j$ i.e. if there is an element which is Eigenelement of Z and which, therefore, is its own successor, the series shall be called finite, otherwise infinite. What the U_i are (i.e. what we count) and particularly whether their series is finite or not, depends on D and Z . If D itself is eigenelement of Z , i.e. if the application of Z does not bring about anything but D itself, we say D is empty with respect to Z . So is the last Element $U_{j'}$ (if there is one): When Z has all "counted away", i.e. when Z generates nothing more, the remainder is empty with respect to Z . Let us call Z^n the repeated application of Z which will lead to U_n . If U_n is a last element, then Z^n is a projection operator, i.e. $(Z^n)^2 = Z^n$. In quantum mechanics projection operators are used to define properties of a system. By analogy, we can say here that Z_n defines a property of D with respect to Z called the cardinality of D .

When counting, we are not obliged to set $D = U_0$. We can also start with $D = U_m$, i.e. we can start counting from m and eventually to the beginnings of life at all. The ordinal number 0, therefore, is not generally distinguished. It just indicates the level chosen as domain of definition for the counting operator in question.

Let us summarize: The beginning of numerical thinking can not be a given set of ordinal numbers M , axiomatically characterised by Peano. It is particularly impossible to define cardinal numbers by means of a metric to be set up in M . This would require the definition of a pair (n, m) of ordinal numbers, i.e., the number two (a pair) has already to be known as a cardinal number. We rather have to start with mentally defined counting operators which generate ordinal numbers. Let Z_m be the counting operator which, when applied to 0, will generate m . So Z_d will generate d . Z_d applied to m may generate n . So we generated three numbers, m, d and n , where d , as easily can be seen, meets all the requirements of a definition of the distance between m and n . Thus, it is counting which generates both the numbers and the metric in the set of ordinal numbers.

This seems to be the same procedure the mental apparatus uses to generate the category of the spatial metric. According to what Piaget (1970, p. 58) found with children, it is not the category of space which allows us to define motion as mapping a line in space to the scale of time. It is rather motion which generates the category of spatial structure. The most primitive intuition, as Piaget called it, (next to the notion of time) is not space but motion. Just as it is impossible to can come from one number to another without a counting (or equivalent) operator, we cannot distinguish points in space except by attributing them to a

path of motion. Counting and moving are analogue terms within the genesis of homologue algebraic and geometrical structures. It is this homology which allows us to extrapolate the observations of motional phenomena in an empirically verifiable manner. The continuity of any physical motion for example is a cognitive phenomenon and not the consequence of an independent law of nature. Formulating discontinuous motions would require a spatial metric which, on the other hand, is only defined by means of the category of motion itself. Discontinuous motions, therefore, can not be realised within the human cognitive apparatus. By this, the degrees of freedom of actual motions are drastically reduced. The same applies for the compactness of numbers we use to establish metric spaces and (regular) analytical functions in metric spaces. Discontinuity of a set of numbers is defined only within the context of a previously defined metric. So, numbers generated by a metric defining (counting) operator are per se compact. Analytical functions in metric spaces are, therefore, born candidates to describe the phenomena of mechanics. This altogether strengthens the assumption that what Davies called the algorithmical compressibility of the world is essentially based upon functional homologies between the mental roots of perceptual and mathematical procedures.

Circularity or the reproduction of the cognitive phenotype

The two major objections against constructivist approaches in cognitive sciences are solipsism and circularity. The solipsistic accusation claims that if one resigns from reality he also must resign from other human beings as parts of reality. What we have done here, however, is not to dispense with the real world as subject of our efforts. We only claim that the results of these efforts are human specific and cannot be described in terms of objective laws of nature.

a) The results of our cognitive efforts are theories which are human specific in so far as they are invariants of human cognitive operators.

b) The results of our physical efforts are survival tools which are human specific in so far they depend on our phylogenetically constructed physical constitution

The difficulty we have to accept the notional character of our experiences as a human specific construct is highly case dependent. Particularly it is different with space and time. As to the notion of space it is undoubted (except perhaps for naïve realists) that the spatial patterns we perceive are not objective in the sense that they could be considered as pictures of real structures, i.e. the world is not necessarily as it looks like visually. Here we concede rather easily to our world picture a reduced objectivity. Not so as to time: The recorded time topology of events we consider to be real. I.e. the order of events is as we perceived it. The past is as it was and even God cannot change it a posteriori we are used to say. Weizsäcker (1985) called this the 'facticity of the past'. Actually, however, events cannot be defined but as results of cognitive or scientific interpretations in a similar way as visual patterns cannot be defined but as invariants of cognitive operators. Events as such are as less defined as visual patterns. A modification of the interpretations used (for example in the light of a novel theory) may well effect the past. There may have been an experiment made in the beginning of this century which documented unambiguously a speed faster than light. After the coming up of the theory of relativity the protocol of the same experiment had to be rewritten in favour of a speed below that of light. Similar would happen if evolution would change our cognitive operators. As this, however, did not happen in historical times the illusion could

arise of both the facticity of the past and the objectivity of the laws of nature.

The allegation that the historicity of the world is a human specific artefact is the more problematic as it is based (via the CEE) just on biological evolution which deals explicitly with the historical order of phylogenetic events. In other words: On the one side our world picture is the construct of our cognitive and experimental apparatus. On the other side, it is just this world picture to which refer physics and biology and the development of our brain including the operators there established. But what, then, is hen and what is egg? Is it the real world we live in and which developed in the course of biotic evolution until the brain functions, or is it just these brain functions (or how ever one should call these roots) which bring about the picture of a real world as a tool for both articulating and solving our problems? In other words: are perceptions part or result of nature or is nature a category brought about by our cognitive apparatus? This dichotomy is the basis of frequent accusations saying that the EE is circular in so far as it would interpret not only the categories of space, time and causality in phylogenetic terms but also the notion of reality and nature, as phylogeny itself is part of nature. Actually, however, there is no real dichotomy as long as it is sure that perceptions and nature condition each other by means of generating each other, i.e.: our cognitive phenotype has to construct a world picture which would permit to understand the genesis of just this cognitive phenotype as evolution within the framework of just this world picture. By this, for the ontogenesis of the cognitive phenotype the same is required as for organic ontogenesis. Both have to be understood as circular, autoreproductive processes:

In the biotic area holds: it is exclusively the epigenetic system of an organism which decides how the genom's structure has to be interpreted and expressed to the phenotype. However, identical reproduction is possible only if the epigenetic system brings about a phenotype comprising the epigenetic system itself.

In the cognitive area holds: It is exclusively the cognitive apparatus (and the science based upon it) which decides how the sensory input has to be interpreted and to which world picture it will be expressed. Consistent and reproducible, however, is such an acquisition of knowledge only if the cognitive/scientific apparatus will bring about a world picture comprising the cognitive/scientific apparatus itself.

Neither can a genome by its own determine the phenotype in the sense of a 'blueprint' - it rather represents one of several levels in the process of autoreproduction - nor can the sensual input determine its own interpretation and by this the reactions it may effect. This does not contradict the fact that within the context of a given organic or cognitive phenotype with a given interpretation machinery, a genetic mutation as well as new perception may lead to reproducible modifications of our physical constitution or of our theories. What we have to avoid, however, is to conclude that what mutations and perceptions will initiate they also will determine. Determinism is possible only within a given scheme of interpretations.

Circularity, a killing objection for any theory within the context of classical realism, will be (in the sense explained here) a necessary prerequisite for any complete constructivistic approach. This constitutes the very difference between realism (of what ever class) and constructivism as presented here. Realism requires of the life mastering methods consistency with an independent outside world. A 'radically' constructivistic interpretation of the world, however, needs only to reconstruct itself. In so far the so called radical constructivism of Foerster, Glasersfeld, Maturana and Varela is not really radical. In order to avoid conflict with the well established term 'radical constructivism' one rather should speak in terms of 'complete constructivism' when characterising what the CEE intends to represent.

Darwinism and CEE have an interesting element in common. Darwinism tries to

explain the actual situation of life as the result of evolution. Evolutionary models which fail in doing so will be victimized by selection. Darwinistic models, so to speak, apply their own content to themselves. Something similar does the CEE. It tries to simulate the evolution of stable forms of life. The main criterion of stable forms is by definition their reproduction capability. The same applies for the CEE. The CEE is stable or 'complete' only if it can reproduce itself in the sense explained here. So the CEE as well applies its own content to itself. Selection and autoreproduction can be considered to be the key terms constituting the Darwinistic and the constructivistic approaches respectively. Both approaches represent metatheories which as such do not make experimentally verifiable statements in which they could contradict each other. Neither excludes the Darwinistic view that the selecting environment is a construct of the organisms concerned themselves nor excludes the CEE that specific constructs can be selected due to lacking coherence with other constructs.

References (6)

- Barrow, J. D. (1990): *Theories of everything*, Oxford University Press
- Campbell, D. T. (1973): *Evolutionary epistemology*. in Schilpp, P. (ed.): *The Philosophy of Karl Popper*. Part I, Open Court, La Salle, pp. 413-463
- Chalmers, A. F. (1982): *What is this thing called science?* Buckingham: Open University Press
- Davies, P. C. W. (1990a): *Why is the physical World so comprehensible?* In *Complexity, Entropy and the Physics of Information*, Santa Fe Institute studies in the Sciences of Complexity, ed. W. H. Zurek, Vol VIII, Addison Wesley, p. 61-70.
- Davies, P. C. W. (1990b): *Why is the universe knowable?* To appear in *Mathematic and Science*, World Scientific Press, ed. Ronald E. Mickens, p. 14-33.
- Dietrich, O. (1989): *Kognitive, organische und gesellschaftliche Evolution*. Berlin, Hamburg: Parey.
- Dietrich, O. (1990): *Sprache als Theorie*. In: *Akten des 6. Int. Kongresses der Deutschen Gesellschaft für Semiotik (DGS)*, Passau, Okt. 1990.
- Dietrich, O. (1992): *Darwin, Lamarck and the Evolution of Life and Culture*. *Evolution and Cognition*, Vol 2, No.3
- Dietrich, O. (1993): *Cognitive and Communicative Development in Reality free Representation*. *Cognitiva* 1993, 5 (2), 219-243
- Dietrich O. (1994a): *Heisenberg and Gödel in the Light of Constructivist Evolutionary Epistemology*. *Ludus Vitalis*, Vol 2, Nr. 2, pp. 119-130
- Dietrich, O. (1994b): *Is There a Theory of Everything?* *Bulletin of the Institute of Mathematics and its Applications*. Vol 80, p. 166-170
- Einstein, A., Bosso, M. (1972): *Correspondance 1903-1955*, Paris.
- Feynman, R. P. (1965): *The character of physical law*. BBC Publication.
- Hawking, S. W. (1979): *Is the end in sight for theoretical physics?* *Inaugural Lecture for the Lucasian Chair*. University of Cambridge.
- Hofstadter, D. R. (1979): *Gödel, Escher, Bach*. New York: Pinguin Books.
- Katz, M. J. (1982): *Ontogenetic Mechanisms: The Middle Ground of Evolution*. In: J. T. Bonner (Hrg.): *Evolution and Development*. Berlin, Heidelberg: Springer.
- Lorenz, K. (1971): *Knowledge, Beliefs and Freedom*. In Weiss, P. (hrg.): *Hierarchically organized systems in theory and practice*. New York: Hafner.
- Lucas, J.R. 1961, *Philosopy*, **36**, 120-124
- Penrose, R. 1989: *The Emperor's new Mind*. Oxford University Press

- Piaget, J. (1967): *Biologie und Erkenntnis*. Frankfurt: S. Fischer
- Piaget, J. (1970): *Genetic Epistemology*. New York: Columbia University Press.
- Popper, R. K. (1973): *Objektive Erkenntnis*. Hamburg: Hoffman und Campe.
- Popper, R. K. (1982): *Logik der Forschung*. Tübingen: J. C. B. Mohr.
- Prigogine, I. (1979): *Vom Sein zum Werden*. München, Zürich: Piper.
- Riedl, R. (1975): *Die Ordnung des Lebendigen*. Berlin: Paul Parey.
- Solomonoff, R. J. (1964): A formal theory of inductive inference. Part I. in *Information and Control*, 7, 1.
- Üxküll, J. von (1921): *Umwelt und Innenleben der Tiere*. Berlin, Springer
- Vollmer, G. (1980): *Evolutionäre Erkenntnistheorie*. Stuttgart: S. Hirzel
- Weizsäcker, C.F. von (1985): *Aufbau der Physik*. München: Hanser
- Wigner, E. (1960): The unreasonable effectiveness of mathematics in the natural sciences. *Comm. Pure Appl. Math.* 13, 1.