The State Property Systems of Contextual Quantum-like Models

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In this paper we adopt an operational approach to quantum mechanics in which a physical entity is determined by the mathematical structure on its set of states Σ, its set of properties \( \mathcal{L} \) and a relation of ‘actuality’ \( \mu \) between these two sets which expresses which properties are actual when the system is in a specific state \([1, 2]\). The State–Property–System (SPS) \((\Sigma, \mathcal{L}, \xi)\) of quantum and classical mechanical entities is such that \( \mathcal{L} \) is a complete orthocomplemented lattice that satisfies the covering law, and is weakly modular and plane transitive \([3, 4]\). The reverse statement is expressed in the representation theorem of Piron \([1]\) who showed that if the set of properties \( \mathcal{L} \) contains at least 4 atoms and is a complete, orthocomplemented, atomistic, weakly modular lattice that satisfies the covering law, and such that the states are represented by the atoms of this lattice, in the infinite dimensional case the standard quantum formalism with classical superselection rules is recovered, but over a ‘generalized Hilbert space’ (also called modular space). Maria Pia Solèr closed the debate by showing that each infinite dimensional modular space that has an orthonormal basis is isomorph with one of the three standard Hilbert spaces \([5]\).

To illustrate this approach, we consider a model in which the maximal change of state of the system due to interaction with the measurement context is controlled by a parameter which corresponds with the number \( N \) of possible outcomes in an experiment. In the limit \( N = 2 \) the system reduces to a model for the spin measurements on a quantum spin-1/2 particle \([6]\). In the other limit \( N \to \infty \) the system is classical, i.e. the experiments are deterministic and its set of properties is a Boolean lattice \([7]\). We show that for intermediate values of \( N \) two axioms used in Piron’s representation theorem are violated, namely the covering law and weak modularity. Next, we discuss a modified version of the model for which it is even impossible to define an orthocomplementation on the set of properties.

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References


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