

Collective Reasoning under Uncertainty and Inconsistency

Martin Adamčík

*School of Mathematics, The University of Manchester,
Oxford Road, M13 9PL, Manchester
martin.adamcik@manchester.ac.uk*

ABSTRACT

Within the framework of discrete probabilistic uncertain reasoning a large literature exists justifying the maximum entropy inference process, **ME**, as being optimal in the context of a single agent whose subjective probabilistic knowledge base is consistent and is represented by a non-empty closed convex set of “possible” probabilistic probability functions. In effect, given such a set of possible probability functions, **ME** “chooses” that unique belief function from the set whose entropy is maximal.

Wilmers [5], Kern-Isberner and Rödder [2], Williamson [6] and others in various ways extended **ME** and inference processes to the context of several agents whose subjective probabilistic knowledge bases, while individually consistent, may be collectively inconsistent, but who seek to merge their collective probabilistic knowledge into a single “social” probability function using objective criteria. Such a process is called a social inference process.

In this talk we will present an approach how to choose an “optimal” social inference process using information theoretic point of view whose origins go back to nineteenth century statistical mechanics [3]. In particular we extend the justification of **ME** from [4] to the multi-agent context.

In effect, by combining cross-entropy updating based on Kullback-Leibler divergence and linear pooling operator of decision theory (cf. [1]) we justify a certain social inference process called “linear entropy process”, **LEP**, which extends **ME** inference process. Moreover **LEP** is closely related to the “social entropy process”, **SEP**, defined by Wilmers in [5]. In fact, from the informational distance point of view (expressed by Kullback-Leibler divergence), **LEP** is dual to **SEP** and both are satisfying some nice principles.

Keywords: Uncertain reasoning, discrete probability function, social inference process, maximum entropy, Kullback-Leibler.

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